The
Mathematical Gazette

A JOURNAL OF THE MATHEMATICAL ASSOCIATION

Vol. 102 July 2018 No. 554

Linear and non-linear models for national health expenditures in the USA

ALLAN J. KROOPNICK

In this brief Article, using the elementary theory of differential equations as well as some basic economic theory, we will develop several estimates for national health expenditures for the United States: one using a linear model and three using non-linear models. We will derive the non-linear models first and then compare them to the linear one in order to see if they differ significantly. While these estimates are for the United States, the methods used here, because they are robust, could be used for any country. Statistical information may be obtained from the World Bank databases which store health statistics by country [1].

What we will do here is estimate the total health costs as a percentage of gross domestic product (GDP) if no further copayments are required. In other words, we are seeking to estimate the total cost of health care as a percentage of GDP when all health care costs are covered by insurance and government subsidy. Several models will be discussed here since such estimates may be made using a variety of assumptions. There is no 'best' model, although such a decision is possible when comparing the estimates to actual data. This is certainly an area where more research is certainly needed. Such issues are important when considering how controversial the Affordable Care Act has been both before and during the last presidential election as well as the cost of a possible replacement for it. What we show here is that reasonable estimates may be obtained using standard methods. Before proceeding, however, we need to introduce several basic economic concepts which are discussed below.

Preliminaries

Let the price P represent the proportion of the total cost of health services that the individual pays directly, usually referred to (in the USA) as 'out of pocket expenses' or coinsurance. P is defined, therefore, between 0 and 1. That is $0 \leqslant P \leqslant 1$. The case $P = 0$ represents 100% or full insurance while the case $P = 1$ represents no insurance. A 10% or 20%

https://doi.org/10.1017/mag.2018.49 Published online by Cambridge University Press

coinsurance would be represented by $P = 0.10$ and $P = 0.20$ respectively. Currently, the Medicare Part B requires a 20% copaymentx for physician services. In general, $P = 0.20$ is consistent with the study by Scitovsky and Snyder [2] which concluded using a 25% coinsurance (i.e., $Q(0.25)$) that doctors' services were utilised at a level of 0.75 and out-patient services at 0.80. Consequently, using the value $P = 0.20$ certainly seems reasonable in the following estimation process.

Next, let $Q(P)$ represent the quantity of health services performed at price P. For $P = 0$, we have a maximum for Q while for $P = 1$ we have a minimum for Q. Moreover, we assume Q is a decreasing function of P because consumers will be less likely to seek tests or treatments if the burden of cost for these services is on them. Thus, the graph of Q as a function of P will be a decreasing curve joining the points $\left(0, Q(0)\right)$ and $\left(1, Q(1)\right)$. Without loss of generality we may set the value of $Q(0)$ to 1. We now discuss the first non-linear model.

First non-linear model

Assume that should P increase by 1%, then Q will decrease by $k \times 100\%$. In general, a small increase in the fraction of health care cost that the consumer must pay will cause a corresponding fractional decrease k times as large as his demand for health care. Thus, $dQ = -kQ\,dP$ or $\frac{dQ}{dP} = -kQ$. This is the well-known differential equation for exponential decay so we conclude that $Q(P) = Q(0)\exp(-kP)$. Setting $P = 1$ and solving for the parameter k yields $k = \log\left(\frac{Q(0)}{Q(1)}\right)$. Now suppose $\frac{Q(0)}{Q(1)} \approx 2$ which some limited historical data seems to support ([2] and [3]). That means the demand for medical services when health insurance pays the bill is twice as great when consumers pay the full cost.

Moreover, this model indicates that if consumers pay about 20% of their health costs then

$$Q(0.20) = Q(0)\exp\left(-0.20\log 2\right) \approx 0.87Q(0). \tag{1}$$

This implies the total demand of health care would be about 13% less than it would be if health care were fully covered.

Second non-linear model

Recall that the dollar cost of health care to the consumer is given by the product PQ. This model assumes that an increase in $d(PQ)$ dollars of medical costs causes a proportional decrease c in demand for medical services. In other words $d(Q) = -c\,d(PQ)$. Using the product rule for differentiation we have $dQ = -c\left(P\,dQ + Q\,dP\right)$. Rearranging terms yields $(1 + cP)\,dQ = -cQ\,dP$. From this, we have the following differential equation:

$$Q'(P) = -\frac{cQ}{1 + cP}. \tag{2}$$

https://doi.org/10.1017/mag.2018.49 Published online by Cambridge University Press

Integrating (2) yields the solution

$$Q(P) = \frac{Q(0)}{1 + cP}.$$

If we again set $\frac{Q(0)}{Q(1)} \approx 2$, then $c \approx 1$. In this instance, $Q(0.20) \approx 0.83Q(0)$. Thus a 20% co-insurance implies a 17% decline in the demand for health care services. This is greater than the estimate for the prior model. The number c in this model is known as the induction factor associated with the differential equation (2). This is called the uniform induction hypothesis in economic theory. This model was in fact used in [4] when estimating the cost of national health insurance.

Third non-linear model

This model is known as the *uniform price elasticity model*. Price elasticity is defined as the ratio of the percentage change in quantity divided by the percentage change in price. If we assume that elasticity is constant then mathematically this means $\frac{\frac{dQ}{Q}}{\frac{dP}{P}} = -b, b > 0$. Rearranging terms yields the differential equation

$$Q'(P) = \frac{-bQ}{P}. \tag{4}$$

The solution to (4) is $Q(P) = kP^{-b}$. However this model is flawed because as $P \to 0$, $Q \to \infty$ which is certainly not allowed. This issue may be resolved by introducing a constant P_i which we can interpret as ancillary expenses such as transport, over the counter medications, etc. Introducing P_i alters the differential equation to

$$Q'(P) = \frac{-bQ}{P + P_i}. \tag{5}$$

The solution to (5) may be written as $Q(P) = Q(0)\left(\frac{P + P_i}{P_i}\right)^{-b}$. For $P = 1$, we have

$$Q(1) = Q(0)\left(\frac{1 + P_i}{P_i}\right)^{-b}. \tag{6}$$

What remains is to evaluate P_i and b. That, however, is no simple matter [3]. From (6) we have

$$b = \frac{\log \frac{Q(0)}{Q(1)}}{\log \frac{1 + P_i}{P_i}}. \tag{7}$$

https://doi.org/10.1017/mag.2018.49 Published online by Cambridge University Press

As before, assume $\frac{Q(0)}{Q(1)} \approx 2$. The next step is to estimate P_i. If we assume that 10% of the expenses are not covered (i.e. $P_i = 0.10$) then from (7) we have $b = 0.289048$. Finally, $Q(0.20) \approx 0.73$ which is well below the other estimates. However, this value is sensitive to the values of P_i and b which are in themselves very difficult to estimate accurately.

The linear model

The linear model is much simpler. In this case, if the fractional decrease is assumed constant then $dQ = -kdP$ or $Q'(P) = -k$ so we must have $Q(P) = 1 - kP$. As before, assuming $\frac{Q(0)}{Q(1)} \approx 2$, we have $k = \frac{1}{2}$. Using this model, relation (1) becomes

$$Q(0.20) = Q(0)\left(1 - 0.5 \times 0.20\right) = 0.90Q(0). \tag{8}$$

Under this model, the demand for health services is about 90% of what it would be under full coverage.

Estimating the total cost for each model as a percentage of GDP

According to the World Bank, the percentage of GDP devoted to medical services is 17.1% [1]. If we assume that 80% of medical services (Medicare Part B typically pays for 80% of physician fees so this is not an unreasonable assumption) are covered by insurance (i.e. $P = 0.20$), then by our previous discussion, model 1 represents a level of demand for medical care of 87%. Had full coverage been available (i.e., $P = 0$) the demand would be $\frac{0.171}{0.87} \approx 0.197 = 19.7\%$. For models 2 and 3 the level of demand would be $\frac{0.171}{0.83} \approx 20.6\%$ and 23.4%. The latter model is clearly out of line. For the linear model, the estimate is $\frac{0.171}{0.90} = 19\%$. While these estimates may seem high, with the exception of model 3, they are not unreasonable when one considers that total health expenditures in 1995 were 13.1% of GDP [1]. That is roughly a 30% increase in the span of 20 years.

Conclusion

Although the analysis here is somewhat rudimentary, we have shown that it is possible to quantify the demand for health services as a function of P as well as give plausible estimates of the maximum cost as a percentage of GDP of health services using these models. Moreover, with the exception of the third non-linear model, these models do give results consistent with each other. Even model 3, by using other values for b and P_i may yield estimates consistent with the others. Furthermore, these models may be improved by better estimating the ratio $\frac{Q(0)}{Q(1)}$ and seeing if the ratio varies

https://doi.org/10.1017/mag.2018.49 Published online by Cambridge University Press

over time. That variation certainly would affect these estimates. In general, given a value of H_i, the cost of health care as a percentage of GDP for a given value of P, the ratio $\frac{H_i}{Q(P)}$ would estimate the full cost of health care as a percentage of GDP for $P = 0$. Finally estimating these values accurately could be a promising starting point for further research when estimating health care expenditures.

Acknowledgement

The author would like to sincerely thank the referee for several valuable suggestions.

References

1. The World Bank, Health expenditure, total (% of GDP), accessed January 2018 at: http://data.worldbank.org/indicator/SH.XPD.TOTL.ZS
2. A. A. Scitovsky and N. M. Snyder, Effect of coinsurance on use of physician services, *Social Security Bulletin*, (June 1972) pp. 3-19.
3. C. L. Trowbridge, Quality-Price relationships in health insurance, Actuarial Note Number 79, *Social Security Administration, DHEW Publication Number* (SSA)73-11507 (November 1972).
4. Analysis of health insurance proposals introduced in the 92nd congress, *Ways and Means Committee Print*, United States Government Printing Office (5270-1176) (August 1972).

10.1017/mag.2018.49

ALLAN J. KROOPNICK
University of Maryland University College, The Graduate School, Adelphi, Maryland 20783 USA
e-mail: *allan.kroopnick@faculty.umuc.edu*

Appeal for more referees

The *Gazette* relies on good peer-refereeing to maintain the standard of the articles accepted. We are always seeking new referees, and are currently very short in some areas. In particular, we do not have enough referees for submissions on Euclidean geometry, mechanics, number theory and analysis, although fresh volunteers are welcome for any subject

I would be delighted to hear from any readers who would be willing to join the refereeing team. If you are not sure what is involved, I can send you details of what the work entails and a form so that you can specify particular areas of expertise. Please email me or write to me at the address on the last page of the journal if you are interested in supporting the journal in this vital way.

Gerry Leversha
g.leversha@btinternet.com
Editor, *The Mathematical Gazette*