(section 8.10) is too brief to say anything of use; furthermore, there are no references either to the work of nonstandard analysts on this subject [**1**], or to Eckhaus' famous rejoinder [**2**]. By the way, a much simpler example of canards is the modest first order non-autonomous equation

$$x' = x - x^3 + \lambda \cos(\varepsilon t),$$

with ε small and λ as a bifurcation parameter (this equation occurs in the theory of optical bistability) [**3**].

In this edition of the book there is much to praise; I am sure the next edition will merit nothing but praise.

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2. W. Eckhaus, Relaxation oscillations including a standard chase on French ducks, in *Asymptotic Analysis II* (F. Verhulst ed., Lecture Notes in Mathematics **985**, Springer-Verlag 1983), 449–494.

3. P. Jung, G. Gray, R. Roy and P. Mandel, Scaling law for dynamical hysteresis, *Phys. Rev. Lett.* **65** (1991), 1873–1876.

Lunardi, A. *Analytic semigroups and optimal regularity in parabolic problems* (Progress in Nonlinear Differential Equations and their Applications Vol. 16, Birkhäuser, Basel, Boston, Berlin 1995) xvii+424pp., 3 7643 5172 1, about £100.

One approach towards a general theory of (nonlinear, inhomogeneous, nonautonomous) parabolic differential equations is to start with the most special (linear, homogeneous, autonomous) case and then to proceed in several stages towards greater generality by using various approximation methods. The theory of one-parameter semigroups of operators arose as an abstract approach to (linear, homogeneous, autonomous) differential equations. Indeed, such semigroups correspond to solutions of the Cauchy problem

$$u'(t) = Au(t), \quad u(0) = u_0,$$

where A is an unbounded linear operator on a Banach space X. The solution is given by $u(t) = T(t)(u_0)$, where $\{T(t): t \geqq 0\}$ is the semigroup generated by A. Typical examples arise when A is an elliptic operator on an open subset Ω of $\mathbb{R}^n$ with suitable boundary conditions and X is a space of functions on Ω. In these examples the semigroup is analytic in the sense that it can be extended to a holomorphic semigroup of operators $T(z)$ defined in a sector given by $|\arg z| < \theta$. This allows one to exploit special properties of analytic semigroups such as the Spectral Mapping Theorem and estimates on $\|AT(t)\|$.

Having established that elliptic operators generate analytic semigroups, one can move to inhomogeneous equations by using the variation-of-constants formula and then to nonautonomous equations by perturbation techniques. These methods can then be extended to the simplest class of nonlinear cases, the semilinear equations of the form

$$u'(t) = Au(t) + f(t, u), \quad u(0) = u_0,$$

where typically A is a second-order elliptic operator and $f(t,u)$ may depend on the first-order derivatives of u. The fully nonlinear case can be put in the same form but now the nonlinear term $f(t,u)$ is of the same order as Au. This makes the analysis substantially more delicate and supplementary conditions have to be imposed even to ensure the existence of solutions.

The aim of this book is to follow this approach towards nonlinear parabolic equations. Where this book differs from existing texts (including the contemporaneous triology of H. Amann) is in seeking classical solutions with continuous or Hölder continuous derivatives by working entirely in spaces of continuous functions instead of L^p spaces. In addition, apart from the usual questions of existence and uniqueness of solutions there is detailed examination of optimal regularity—if the input of the equation is sufficiently smooth, then the solution should also be smooth. The author includes abstract theory when it serves her purpose, but not just for the sake of it. Thus she goes directly to the theory of analytic semigroups without discussing the general theory of strongly continuous semigroups and she includes some abstract interpolation theory but only enough to enable her to deal efficiently with Hölder continuous solutions of differential equations.

There are far too many variations of parabolic equations to allow them all to be treated in a single monograph. Rather than attempting to outline all the possibilities the author has been selective in deciding which problems to consider, but she has then covered those cases in detail. For example, she mostly considers only second-order elliptic operators, but she chooses methods which can be generalised to higher-order operators and she gives appropriate bibliographic references. Nevertheless, the details involve many complications both in the statements and in the analytic estimates which are the basis of most of the proofs. Consequently, the book is not a light read except for a few pages of introduction to each chapter. A novice in the subject would probably prefer to start with a simpler text such as A. Pazy's *Semigroups of linear operators and applications to partial differential equations* (2nd edn, Springer-Verlag 1992), but Lunardi's book will become a very valuable reference for anyone working seriously in the area.

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