

Newtonian-like behaviour of starting vortex flow in superfluid helium at high Reynolds numbers

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We study experimentally the starting vortices shed by airfoils accelerating uniformly from rest in superfluid helium-4 (He II). The vortices behave apparently as if they were moving in a classical Newtonian fluid, such as air or water. Specifically, the starting vortex positions obtained from the experimental data are found to be very close to those computed numerically in a Newtonian fluid, at sufficiently small times, when self-similar behaviour is expected to occur, and for Reynolds numbers ranging between approximately 5×10^2 and 5×10^5 . The result indicates neatly that turbulent flows of He II can be very similar to classical flows of Newtonian fluids, when thermal effects can be neglected and at sufficiently large flow scales, i.e. the study demonstrates that He II could also be employed to study classical Newtonian flows.

Key words: quantum fluids, turbulent flows, vortex flows

1. Introduction

Superfluid helium-4, also known as He II, is a remarkable cryogenic liquid, existing solely at temperatures lower than approximately 2 K, in close proximity to absolute zero (Mongiovì, Jou & Sciacca 2018; Barenghi, Skrbek & Sreenivasan 2024). It is characterised by very small values of kinematic viscosity, which can be up to two orders of magnitude smaller than those of water (Donnelly & Barenghi 1998). This in essence means that fully

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developed turbulent flows, which are often associated with high Reynolds numbers, can be achieved in relatively small experimental facilities, as has been demonstrated recently in the case of turbulent vortex rings (Švančara & La Mantia 2022). However, the liquid behaviour does not always resemble that of classical Newtonian fluids, such as air or water. Indeed, as discussed, for example, by Švančara & La Mantia (2019), noticeable differences are seen when heat transport becomes non-negligible and at sufficiently small flow scales, smaller than the mean distance ℓ between the topological defects embedded in the fluid, which are named quantised (or quantum) vortices (Donnelly 1991). These vortices, which have an atomic-size core, can be of macroscopic length, i.e. they can be seen as line-like objects within the superfluid. More importantly, they are held responsible for the unique behaviour of the liquid: for example, the fact that, at sufficiently large fluid velocities, the thermal conductivity of He II depends nonlinearly on the heat flux can be related to the dynamics of quantum vortices (Mongiovì *et al.* 2018). The same can be said of the peculiar properties observed in some thermally driven flows of He II, such as those in channels (Varga, Babuin & Skrbek 2015) and turbulent jets (Švančara, Roche & La Mantia 2023; Obara *et al.* 2025), while other flows, e.g. macroscopic vortex rings (Švančara, Pavelka & La Mantia 2020), display distinctive Newtonian-like features also when thermally generated, i.e. the collective dynamics of quantum vortices does not result solely in non-classical characteristics when driven thermally. Instead, mechanically generated flows of He II display more often Newtonian-like properties at large scales, including, for example, the famous $-5/3$ energy spectrum (Maurer & Tabeling 1998; Baggaley 2012), the four-fifths law of turbulence (Salort *et al.* 2012) and intermittency, at least in some cases (Rusaouen *et al.* 2017; Verma *et al.* 2023) – the reader is referred to Barenghi *et al.* (2024) for a discussion on relevant works. Here we give our contribution to this active field of scientific research, focusing on the comparison between classical and superfluid turbulence, and present an experimental investigation that was performed in He II, in flow conditions close to those associated with Newtonian-like behaviour in previous studies, i.e. in the absence of significant heat transport and at flow scales appreciably larger than ℓ . Our long-term aim is to identify clear ranges of experimental parameters in which Newtonian-like features are observed in flows of He II. Such ranges of experimental parameters could be instrumental in understanding why similarities between turbulent flows of He II and analogous flows of Newtonian fluids exist in the first place, which is unknown at present.

We specifically study the starting vortices generated by airfoils accelerating uniformly from rest in He II using the particle tracking velocimetry technique, already employed in previous studies, such as Švančara *et al.* (2021) and Sakaki, Maruyama & Tsuji (2022). Relatively small solid particles, suspended in the fluid, are illuminated by a thin laser sheet and their flow-induced motions are captured by a digital camera. Then, from the positions and velocities of the particles, the vortices shed at the trailing edges of the airfoils are identified, and their relative strength is quantified using the Lagrangian pseudovorticity, a method that has already been applied to the study of large-scale vortex rings propagating in He II (Švančara *et al.* 2020; Outrata *et al.* 2021). The starting vortex trajectories are compared with those obtained numerically in a Newtonian fluid, for a flat plate accelerating uniformly from rest (Xu & Nitsche 2015; Xu, Nitsche & Krasny 2017) and no significant differences are observed at relatively early times. Similarly, a satisfactory agreement is obtained from the comparison with an analytical theory, valid for an inviscid fluid (Pullin 1978; Pullin & Jane Wang 2004), but only at smaller times. That is, the agreement between experimental and numerical results is more apparent than that between experimental and analytical results. Consequently, the studied flows of He II display neat Newtonian-like features in the range of investigated parameters, i.e. they appear as

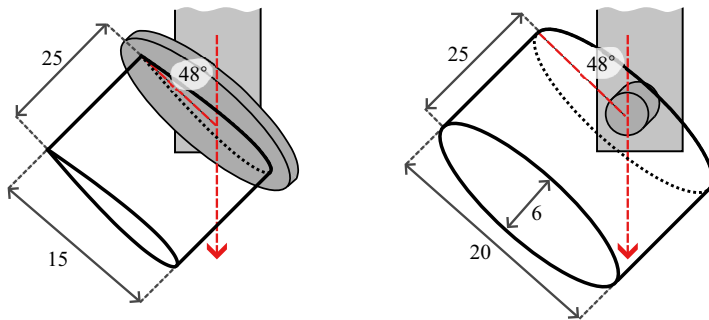


Figure 1. Schematic views of the rectangular wings employed in the study, to scale; dimensions are in mm. The wing on the left-hand side has a NACA 0012 cross-section, while that on the right-hand side has an elliptical profile. The red arrows indicate the motion direction, and the angle of attack (48°) is between the latter and the wing chord. The grey supports on the far sides of the wings link the latter to the linear motor placed outside the cryostat (they are approximately 3 mm thick along the span direction). The laser sheet is located at the wing mid-span, in the middle of our experimental volume – see Blaha (2022), Blaha & La Mantia (2024) and Blaha (2024) for further technical details.

if they were constrained mainly by viscosity, although this behaviour originates most likely from the small-scale dynamics of the quantum vortices embedded in the liquid, as suggested also by recent numerical simulations of macroscopic vortex rings (Galantucci, Krstulovic & Barenghi 2023).

It is now important to remark that we chose to investigate the starting flow problem in He II because it has a rich history in Newtonian fluid mechanics, dating back to the beginning of aerodynamic theory; see Reijtenbagh, Tummers & Westerweel (2023) and Sader *et al.* (2024) for recent examples and references to some early works. It was specifically recognised that solutions of this problem could be relevant for airfoil design and, over the years, the focus has been especially on the estimate of the forces acting on bodies accelerating from rest in a still Newtonian fluid – these forces are in general significantly larger than those resulting from the steady motion of the same objects, as discussed, for example, by Reijtenbagh *et al.* (2023). In other words, we chose this classical problem in view of assessing the behaviour of He II in analogous flow conditions, which were not investigated extensively in the past, although some attention has been devoted to its steady counterpart (Kitchens *et al.* 1965; Musser *et al.* 2019).

2. Methods

The experiments were performed in Prague, at the cryogenic visualisation laboratory of Charles University, already described in previous works (e.g. Švančara *et al.* 2021). Figure 1 shows schematic views of the two rectangular wings employed in this study. The wing on the left-hand side of the figure has a NACA 0012 profile, which is well known in aerodynamics (see e.g. Abbott & von Doenhoff 1959), with chord $L = 15$ mm. That on the right-hand side has an elliptical cross-section, with chord $L = 20$ mm and 6 mm thick. Both wings are made of PMMA and are 25 mm in span. The imposed acceleration was in both cases perpendicular to the wing span, in the vertical direction, and the angle between the motion direction and the wing chord, i.e. the angle of attack, was equal to 48° . The camera field of view was parallel to the wing cross-section, i.e. to the wing vertical motion, and the thin laser sheet was located at the mid-span, to reduce three-dimensional effects. The wing underwent an oscillatory motion. It first moved downward at a constant

	a (m s ⁻²)	T (s)	$Re \times 10^{-3}$	t_a	R
AE	0.17	0.34	116.7	1.46	1.24
BE	0.66	0.17	230.6	1.44	3.20
CE	2.59	0.09	455.2	1.42	8.31
AN	0.16	0.31	73.2	1.63	1.01
BN	0.63	0.15	146.3	1.63	2.72
CN	2.53	0.08	292.4	1.62	7.30
BF	0.64	0.18	0.5	1.41	—

Table 1. Motion parameters. The motion type (first column) is indicated by two letters: the first one is associated with the acceleration magnitude a , with increasing value, from A to C, and the second one with the cross-sectional shape, with E, N and F denoting ellipse, NACA 0012 and flat plate, respectively (for the direct numerical simulation $L = 20$ mm). T indicates the reference time; Re is the Reynolds number; t_a denotes the dimensionless time, in units of T , when the maximum profile velocity is reached, at the set acceleration; R indicates the ratio between estimates of the smallest scale probed experimentally and the mean distance between quantum vortices (see § 3 for details).

acceleration, reached a set velocity and then decelerated to zero velocity. After that, it moved upward in the same fashion, and came back to the initial position. However, we are here interested in the first quarter of the cycle, when the wing is moving at constant acceleration. For each value of acceleration a , approximately 100 movies (cycles) were collected and at least 0.5 (up to 8) million particle positions (and velocities) were obtained from the movies. The data sets associated with the NACA wing are larger than those associated with the elliptical one, i.e. the former have at least 2.5 million particle positions, while the latter up to 1 million. Further technical details are reported by Blaha (2022) and Blaha & La Mantia (2024) for the experiments with the elliptical profile, and by Blaha (2024) for the NACA section experiments.

In both cases experiments were performed at constant temperature, between 1.3 and 2.1 K, to avoid thermal effects, and the starting vortex trajectories were found not to depend appreciably on the fluid temperature (Blaha 2022; Blaha 2024; Blaha & La Mantia 2024). Therefore, the data sets obtained at different temperatures, using the same wing and at the same acceleration magnitude, were later combined and here we do not distinguish between them, considering especially that the kinematic viscosity of He II does not change significantly in this temperature range (Donnelly & Barenghi 1998). Following Xu & Nitsche (2015), we define the Reynolds number as

$$Re = \frac{L^2}{\nu T}, \quad (2.1)$$

where the reference time is

$$T = \left(\frac{L}{a}\right)^{1/2}. \quad (2.2)$$

The kinematic viscosity ν of He II is set here to 10^{-8} m² s⁻¹, for the sake of comparison. The Reynolds numbers obtained in He II are then of the order of 10^5 (see table 1). To get the same Re in water, one would need, for example, 10 times larger chord and acceleration – note that in the direct numerical simulation the fluid viscosity is set to 5×10^{-6} m² s⁻¹, resulting in $Re \approx 500$.

As mentioned above, we estimate the starting vortex strength using the Lagrangian pseudovorticity, i.e. from the particles' positions and velocities, following Švančara *et al.*

(2020) and Oustrata *et al.* (2021). This scalar quantity is defined here as

$$p(\mathbf{r}, t) = \left\langle \frac{(\mathbf{r}_i - \mathbf{r}) \times \mathbf{u}_i}{|\mathbf{r}_i - \mathbf{r}|^2} \right\rangle, \quad (2.3)$$

where $\mathbf{r}_i$ and $\mathbf{u}_i$ indicate the particle positions and velocities, respectively. The angle brackets denote the ensemble average within a set of Lagrangian particles, captured within a time window centred at time t and within an annular region centred, on a chosen grid, at the inspection point $\mathbf{r}$ (the observed particle tracks lie on a plane). The size of the annular region was chosen in order to have enough particles for the calculation of p and, at the same time, to exclude contributions from particles too close to $\mathbf{r}$ – see e.g. Blaha (2024) for details.

It can be shown that the pseudovorticity p is equal to half of the flow vorticity under ideal conditions, which are at present not met in experiments, i.e. one would need many more particles in each image to reconstruct quantitatively the vorticity field (Švančara *et al.* 2020). Consequently, the pseudovorticity is useful to compare quantitatively different flows in the absence of Eulerian data, which is currently the case for He II flows (the vorticity is by definition an Eulerian quantity). Numerical simulations showed that, in the case of vortex rings, the pseudovorticity can track faithfully the ring trajectory, although its magnitude can be significantly smaller than that of the corresponding flow vorticity. The latter was specifically observed when particles' spatial distributions typical of experiments were employed in the simulations, while particles' distributions significantly denser than those used routinely in experiments led to pseudovorticity values much closer to the vorticity ones – see Oustrata *et al.* (2021) for details.

3. Results

In figure 2 we plot the pseudovorticity and vorticity maps obtained at different dimensionless times, for three motion types. The pseudovorticity values reported here are computed as discussed by Blaha (2024), using the parameters there stated also for the ellipse motion types, for the sake of comparison. The vorticity is computed numerically, using a fourth-order finite-difference method, devised specifically to resolve the starting vortex emergence and evolution at early times; see Xu & Nitsche (2015) and Xu *et al.* (2017) for technical details. From the figure it is evident that the size of the starting vortices is strongly influenced by the profile shape, while their location seems similar. Consequently, to investigate further the problem, in a more quantitative way, at least another step is required.

That is, within the obtained pseudovorticity maps we track the positions of the starting vortices shed at the profiles' trailing edges as a function of time, following the procedure described by Blaha (2024), i.e. from the pseudovorticity local maxima. The resulting positions of the starting vortices are plotted in figure 3, in the accelerating reference frame; for the motion type BF (direct numerical simulation) the vortex positions were estimated using streaklines, which were also computed in previous studies (Xu & Nitsche 2015; Xu *et al.* 2017).

At first sight, it is apparent that, once normalised, the vortices' trajectories depend solely on the profile shape. Additionally, the vortices shed from sharp edges move faster than those shed from the elliptical profile. The result could be explained considering that, in a viscous fluid, velocity gradients, which can be associated in a broad sense with energy injection into the fluid, should be more significant at a sharper edge, in comparison with a smoother one, as discussed, for example, by Cadot *et al.* (1997).

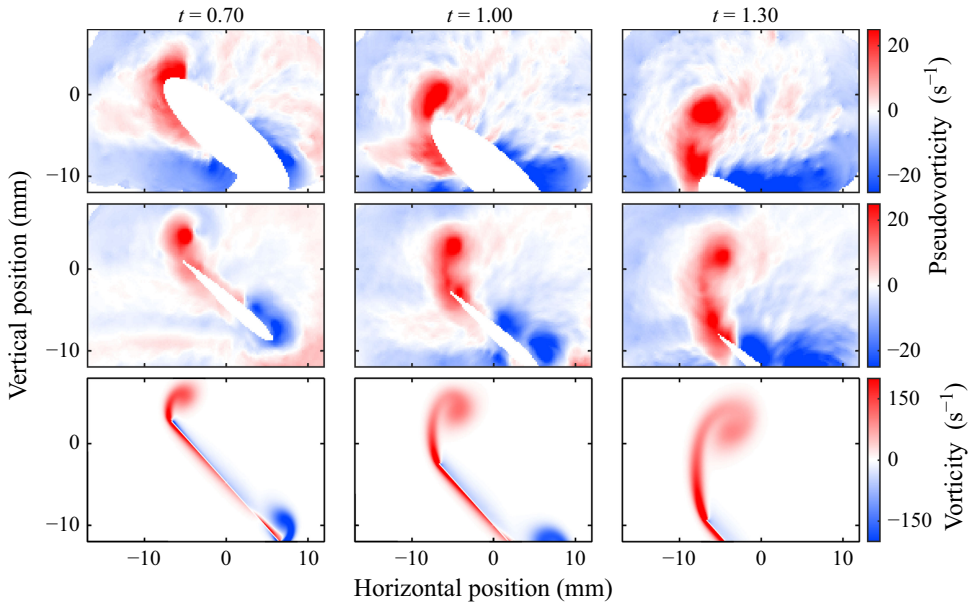


Figure 2. Pseudovorticity and vorticity maps at three dimensionless times, in units of T , for the motion type BE (top row, experiment), BN (middle row, experiment) and BF (bottom row, direct numerical simulation); relevant parameters are given in [table 1](#). The maps are plotted in the laboratory frame, with origin at the section mid-chord, at $t = 0$ (at later times the wing is moving downward, in the vertical direction, at the given acceleration magnitude). The angle of attack (48°) is between the vertical and chord directions. The trailing-edge vortex is seen on the left-hand side of the maps.

We observe a remarkable agreement between the experimental and numerical results, indicating strongly that the flows under investigation are occurring apparently in a Newtonian fluid, at least in the range of parameters here considered. Indeed, at sufficiently small times, the vortex trajectories associated with the flat plate (BF) and with the NACA section (AN, BN, CN) are almost indistinguishable – both profiles have a trailing edge sharper than that of the elliptical section. The closer agreement observed at later times between the trajectories associated with the flat plate (BF) and with the elliptical profile (AE, BE, CE) can be explained considering that, at these late times, the sizes of the starting vortex and section become comparable, i.e. one could say that the symmetry of the flat plate and elliptical profile is seen by the vortex (for the NACA section the leading edge is less sharp than the trailing one). More generally, the worsening of the agreement at later times, between the numerical and experimental results, might be due to three-dimensional, finite-size effects, which are not considered in the numerical scheme. Note also that, as shown in [table 1](#), the Reynolds number of the direct numerical simulation is much smaller than those of the experiments, i.e. our results indicate that, for the studied accelerating motions, Reynolds-number effects should be negligible – this is consistent with the numerical results reported by Sader *et al.* (2024) for different types of starting flows.

We also compare the numerical and experimental results with those obtained from an inviscid theory (Pullin & Jane Wang 2004) considering a flat plate accelerating uniformly from rest with the given angle of attack – corresponding positions are indicated with the red line in [figure 3](#). The agreement between the vortex positions is apparent only at relatively small dimensionless times, up to approximately $T/2$. The result can be associated with the starting vortex self-similar growth, when the vortex size is small compared with the profile chord; at later times, when the vortex has grown into a size such

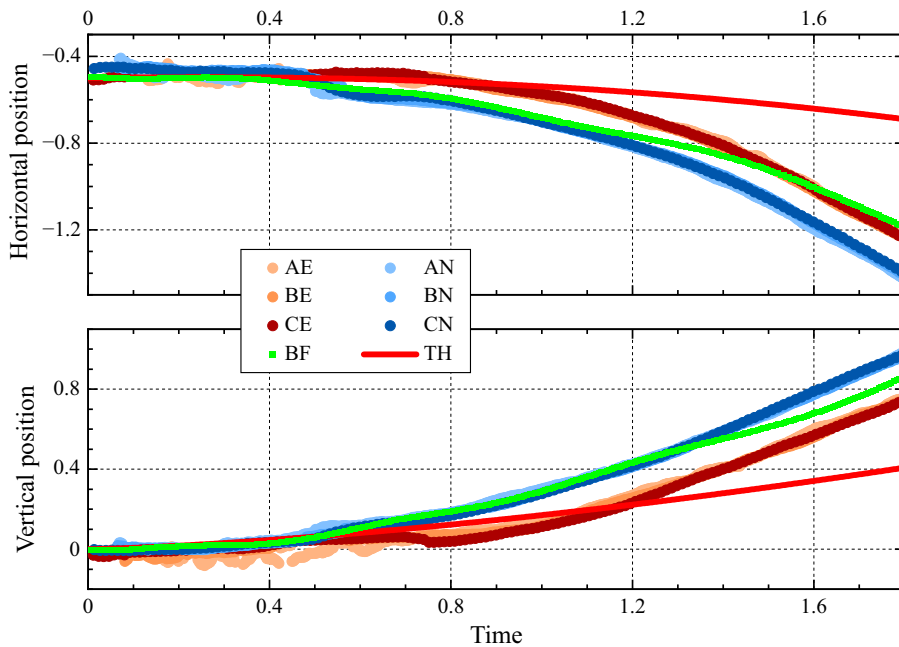


Figure 3. Dimensionless positions, in units of L , of the trailing-edge vortex as a function of dimensionless time, in units of T , in the accelerating reference system, with origin at the profile mid-chord, vertical axis perpendicular to the chord, pointing upward, and horizontal axis parallel to the chord, pointing to the right (leading edge); symbols as in table 1. Here TH indicates the positions computed using an inviscid theory (Pullin & Jane Wang 2004) considering a flat plate accelerating uniformly from rest with the chosen angle of attack.

that the finite chord of the section is no longer negligible, the inviscid model is not expected to be applicable, as discussed, for example, by Xu & Nitsche (2015). Specifically, it has been reported that the starting vortex flow follows well the analytical self-similar theory only at some distance away from the edge (and at sufficiently small times), while viscous effects are found to be significant mainly in close proximity to the wing, i.e. within the boundary layer, which was not accessed in the present experiments. For instance, Pullin & Perry (1980) conducted relevant experiments in water and compared satisfactorily the obtained results with those of an inviscid fluid theory (Pullin 1978) – see Sader *et al.* (2024) for other relevant examples.

Indeed, as shown in figure 4, the agreement between the numerical and inviscid theory results is more evident at the smallest times, when self-similar behaviour is expected to occur. It is also apparent from the figure that, at these small times, the vortex positions identified from the experimental data are more scattered than those computed from the inviscid theory and the numerical data, although the overall trends are consistent with each other, especially at the highest accelerations. To improve the situation, one would likely need many more particles in each image, to identify more neatly the pseudovorticity local maxima in the proximity of the wing. Note, in this regard, that the positions associated with the elliptical profile are more scattered than those related to the NACA section, which were obtained using significantly larger data sets, as noted above.

It is now useful to say that we also performed similar experiments in a Newtonian fluid, using the same NACA wing and the same visualisation technique. Specifically, we chose parameters close to those associated with the motion type BN; see table 1. The main difference is the Reynolds number, i.e. these experiments were performed in

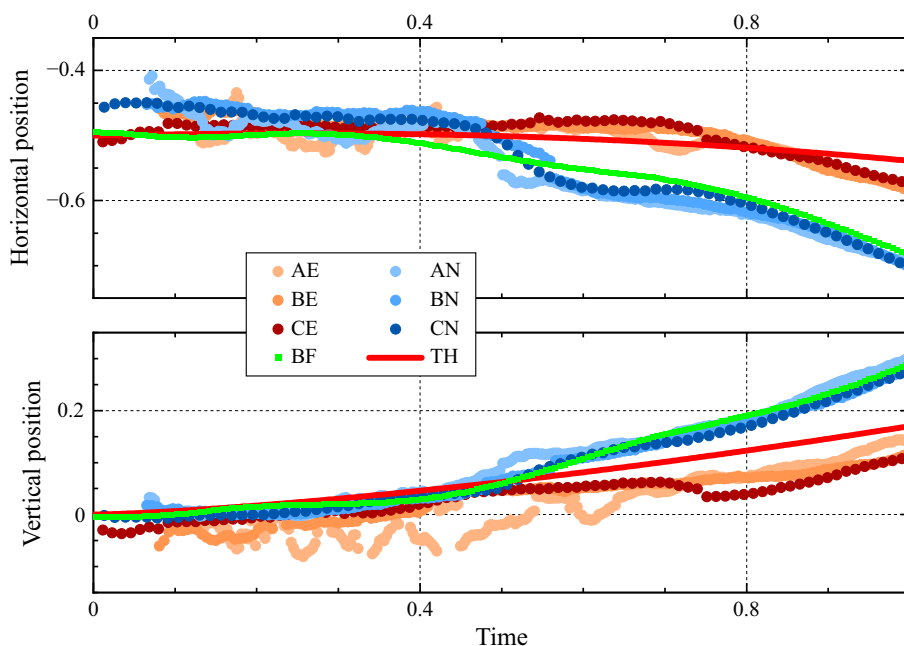


Figure 4. Enlargement of [figure 3](#) at early times; symbols as in [figure 3](#).

liquid helium-4, at 2.3 K, which is a classical Newtonian fluid, called helium I or He I. This liquid is characterised by relatively small values of kinematic viscosity, up to three times higher than those of He II (Donnelly & Barenghi 1998), and by values of thermal conductivity that can be orders of magnitude lower than those of He II (Mongiovì *et al.* 2018) – note in passing that He I becomes He II (superfluid helium-4) at approximately 2.2 K (Donnelly & Barenghi 1998). The number of particle positions (and velocities) obtained in He I was approximately five times smaller than in He II, because fewer movies were collected for these parameters. The resulting pseudovorticity maps are consequently more scattered in He I than in He II, as is apparent in [figure 5](#). It also follows that the vortex tracking algorithm employed here (Blaha 2024) is not suitable for such scatter data sets. However, from these maps one can still see that the location of the starting vortex looks similar in these two fluids, as in [figure 2](#). Additionally, we performed more recently another series of experiments, using the same wing and visualisation technique, both in He I and in He II, with analogous motion parameters, and no appreciable differences are observed in the starting vortex trajectories. In this more recent experimental campaign we collected more data in He I, in comparison with the present experiments, but the angle of attack was slightly larger (49°) and three-dimensional effects were more evident – see Brichet (2024) for preliminary results.

In summary, our main outcome, shown in [figure 3](#), indicates clearly that the flows of He II here investigated behave as if they were flows of a Newtonian fluid. To substantiate further the claim, it is useful to mention that numerical results obtained employing the same computational techniques have already shown agreement with relevant experimental observations, as discussed, for example, by Xu & Nitsche (2015). Similarly, it was already shown that, under some conditions, experimental results obtained in Newtonian fluids cannot be distinguished from those obtained in He II employing the same investigative tools, as demonstrated, for example, by Maurer & Tabeling (1998) and Švančara & La Mantia (2017). In the same spirit of these studies, we now estimate the mean distance

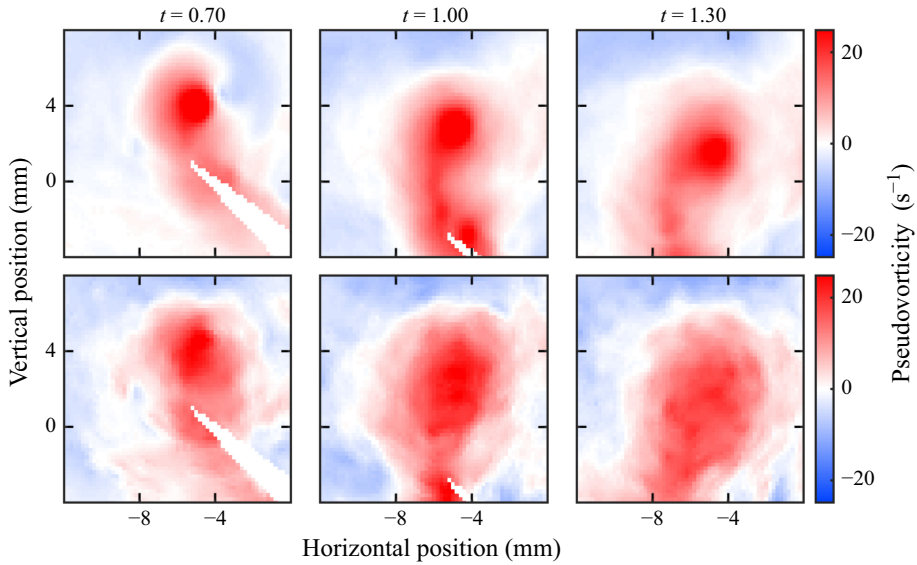


Figure 5. Top row: pseudovorticity maps at three dimensionless times, in units of T , for the motion type BN; see [table 1](#) for relevant parameters. Bottom row: pseudovorticity maps at three dimensionless times, as in the top panels, for a motion type similar to BN, but occurring in a Newtonian fluid (He I), with $a = 0.63 \text{ m s}^{-1}$, $T = 0.15 \text{ s}$, $Re = 73 \times 10^3$ – the corresponding kinematic viscosity ν is equal to $2 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ (Donnelly & Barenghi 1998) – and $t_a = 1.62$. The maps are plotted in the laboratory frame, as in [figure 2](#), but the extrema of the axes are smaller than those of [figure 2](#), to highlight the starting vortex location.

ℓ between quantum vortices to show that here we are actually probing flow scales significantly larger than ℓ because, as noted above, we do expect Newtonian-like features to occur at these large flow scales. Specifically, in the absence of significant thermal effects, as in the present case, one can write, following, for example, Blaha (2024),

$$\ell = \left(\frac{\kappa}{\omega} \right)^{1/2}, \quad (3.1)$$

where $\kappa = 9.97 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ is the quantum of circulation (Donnelly & Barenghi 1998), associated with each quantised vortex, and ω denotes the flow vorticity. Additionally, as noted above, it was shown that the pseudovorticity is equal to half of the vorticity under conditions that are at present not met in experiments (Švančara *et al.* 2020). Consequently, to obtain an upper-bound estimate of the mean distance between quantum vortices, we set

$$\ell = \left(\frac{\kappa}{2\bar{p}} \right)^{1/2}, \quad (3.2)$$

where $\bar{p}$ indicates the average value of the positive pseudovorticity, associated with the trailing-edge vortex and computed from the experimental data, for dimensionless times smaller than 1.8, which is consistent with the time range displayed in [figure 3](#). Similarly, an estimate of the smallest scale probed by our particles can be obtained multiplying the maximum profile velocity, reached at the dimensionless time t_a reported in [table 1](#), and the time between subsequent particle positions, used for the particles' velocity estimate (0.002 s). The ratio R between the latter and the estimate of the mean distance between quantum vortices is listed in [table 1](#). Considering that, as already noted, in the case of vortex rings (Oustrata *et al.* 2021) the pseudovorticity can be significantly smaller than the vorticity, we can safely say that in the present case our particles are probing flow scales

significantly larger than the mean distance between quantum vortices. Note, in this regard, that the magnitudes of the vorticity maxima obtained in the direct numerical simulation (BF) are significantly larger than those in the pseudovorticity maps, as shown in [figure 2](#).

4. Conclusions

The results reported in the present work demonstrate striking similarities between starting vortex flows in He II and in a Newtonian fluid, at sufficiently large flow scales, larger than the mean distance between quantum vortices, and in the absence of significant thermal effects. Is this just a coincidence, or could quantitative relations exist between the collective behaviour of quantised vortices and viscosity effects? We believe that further studies are required to clarify the issue. For example, future investigations on starting vortex flows should be devoted to the collection of more data not only in He II but also in Newtonian fluids, especially in view of comparing the shape and strength of the shed vortices. More generally, we believe that the quantitative identification of ranges of experimental parameters in which Newtonian-like features are observed in flows of He II could be instrumental in understanding why these similarities exist in the first place. On the other hand, these similarities could enable us to investigate classical flows directly using He II, within the identified parameter ranges, taking especially advantage of the extremely small kinematic viscosity of this unique liquid, resulting in relatively small experimental facilities, significantly smaller than typical wind tunnels. Indeed, the latter have cross-sectional areas of $O(10^4)$ cm², while the cross-section of the cryogenic wind tunnel employed in the present experiments is 25 cm² (Blaha [2024](#)).

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Declaration of interests. The authors report no conflict of interest.

Data availability statement. The data that support the findings of this study are available from the corresponding author upon reasonable request.

Author contributions. M.L.M. designed the research. Experiments were performed in Prague by J.B. and M.L.M. Numerical simulations were performed by L.X. All authors contributed to data processing and interpretation, and to manuscript preparation.

REFERENCES

- ABBOTT, I.H. & VON DOENHOFF, A.E. 1959 *Theory of Wing Sections*. Dover Publications.
- BAGGALEY, A.W. 2012 The importance of vortex bundles in quantum turbulence at absolute zero. *Phys. Fluids* **24** (5), 055109.
- BARENGHI, C.F., SKRBEK, L. & SREENIVASAN, K.R. 2024 *Quantum Turbulence*. Cambridge University Press.
- BLAHA, J. 2022 Motion of a large cylinder in superfluid helium. Bachelor's thesis, Charles University.
- BLAHA, J. 2024 Starting flow past an accelerating body in superfluid helium. Master's thesis, Charles University.
- BLAHA, J. & LA MANTIA, M. 2024 Visualization of the starting vortex shed by a cylinder accelerating in superfluid helium-4: preliminary results. In *Proceedings of the Conference Topical Problems of Fluid Mechanics*, pp. 15–22. Institute of Thermomechanics, Czech Academy of Sciences.
- BRICHET, L. 2024 Experimental investigation of vortex shedding in superfluid ⁴He. Internship Report. Charles University. [arXiv.2501.19292](#).

- CADOT, O., COUDER, Y., DAERR, A., DOUADY, S. & TSINOBER, A. 1997 Energy injection in closed turbulent flows: stirring through boundary layers versus inertial stirring. *Phys. Rev. E* **56** (1), 427–433.
- DONNELLY, R.J. 1991 *Quantized Vortices in Helium II*. Cambridge University Press.
- DONNELLY, R.J. & BARENGHI, C.F. 1998 The observed properties of liquid helium at the saturated vapor pressure. *J. Phys. Chem. Ref. Data* **27** (6), 1217–1274.
- GALANTUCCI, L., KRSTULOVIC, G. & BARENGHI, C.F. 2023 Friction-enhanced lifetime of bundled quantum vortices. *Phys. Rev. Fluids* **8** (1), 014702.
- KITCHENS, T.A., STEYERT, W.A., TAYLOR, R.D. & CRAIG, P.P. 1965 Flow visualization in He II: direct observation of Helmholtz flow. *Phys. Rev. Lett.* **14** (23), 942–945.
- MAURER, J. & TABELING, P. 1998 Local investigation of superfluid turbulence. *Europhys. Lett.* **43** (1), 29–34.
- MONGIOVI, M.S., JOU, D. & SCIACCA, M. 2018 Non-equilibrium thermodynamics, heat transport and thermal waves in laminar and turbulent superfluid helium. *Phys. Rep.* **726**, 1–71.
- MUSSER, S., PROMENT, D., ONORATO, M. & IRVINE, W.T.M. 2019 Starting flow past an airfoil and its acquired lift in a superfluid. *Phys. Rev. Lett.* **123** (15), 154502.
- OBARA, K., KENTO, K., YOSHISAKA, S. & JANO, H. 2025 Quantum turbulence triggered by counterflow in a connecting tube. *J. Low Temp. Phys.* **218** (3–4), 136–145.
- OUTRATA, O., PAVELKA, M., HRON, J., LA MANTIA, M., POLANCO, J.I. & KRSTULOVIC, G. 2021 On the determination of vortex ring vorticity using Lagrangian particles. *J. Fluid Mech.* **924**, A44.
- PULLIN, D.I. 1978 The large-scale structure of unsteady self-similar rolled-up vortex sheets. *J. Fluid Mech.* **88** (3), 401–430.
- PULLIN, D.I. & JANE WANG, Z. 2004 Unsteady forces on an accelerating plate and application to hovering insect flight. *J. Fluid Mech.* **509**, 1–21.
- PULLIN, D.I. & PERRY, A.E. 1980 Some flow visualization experiments on the starting vortex. *J. Fluid Mech.* **97** (2), 239–255.
- REIJTENBAGH, J., TUMMERS, M.J. & WESTERWEEL, J. 2023 Drag force on a starting plate scales with the square root of acceleration. *Phys. Rev. Lett.* **130** (17), 174001.
- RUSAOUEN, E., CHABAUD, B., SALORT, J. & ROCHE, P.-E. 2017 Intermittency of quantum turbulence with superfluid fractions from 0 % to 96 %. *Phys. Fluids* **29** (10), 105108.
- SADER, J.E., HOU, W., HINTON, E.M., PULLIN, D.I. & COLONIUS, T. 2024 The starting vortices generated by bodies with sharp and straight edges in a viscous fluid. *J. Fluid Mech.* **992**, A15.
- SAKAKI, N., MARUYAMA, T. & TSUJI, Y. 2022 Study on the curvature of Lagrangian trajectories in thermal counterflow. *J. Low Temp. Phys.* **208** (3–4), 223–238.
- SALORT, J., CHABAUD, B., LÉVÊQUE, E. & ROCHE, P.-E. 2012 Energy cascade and the four-fifths law in superfluid turbulence. *Europhys. Lett.* **97** (3), 34006.
- ŠVANČARA, P. *et al.* 2021 Ubiquity of particle-vortex interactions in turbulent counterflow of superfluid helium. *J. Fluid Mech.* **911**, A8.
- ŠVANČARA, P. & LA MANTIA, M. 2017 Flows of liquid ^4He due to oscillating grids. *J. Fluid Mech.* **832**, 578–599.
- ŠVANČARA, P. & LA MANTIA, M. 2019 Flight-crash events in superfluid turbulence. *J. Fluid Mech.* **876**, R2.
- ŠVANČARA, P. & LA MANTIA, M. 2022 Coherent propagation of vortex rings at extremely high Reynolds numbers. *J. Fluid Mech.* **953**, A28.
- ŠVANČARA, P., PAVELKA, M. & LA MANTIA, M. 2020 An experimental study of turbulent vortex rings in superfluid ^4He . *J. Fluid Mech.* **889**, A24.
- ŠVANČARA, P., ROCHE, P.-E. & LA MANTIA, M. 2023 Spatial distribution of the quantized vortices’ tangle in thermally driven round jets of superfluid helium. *Phys. Fluids* **35** (3), 035136.
- VARGA, E., BABUIN, S. & SKRBEK, L. 2015 Second-sound studies of coflow and counterflow of superfluid ^4He in channels. *Phys. Fluids* **27** (6), 065101.
- VERMA, A.K., SHUKLA, S., SHUKLA, V., BASU, A. & PANDIT, R. 2023 Statistical properties of superfluid turbulence in ^4He from the Hall-Vinen-Bekharevich-Khalatnikov model. *Phys. Rev. E* **108** (4), 045103.
- XU, L. & NITSCHKE, M. 2015 Start-up vortex flow past an accelerated flat plate. *Phys. Fluids* **25** (3), 033602.
- XU, L., NITSCHKE, M. & KRASNY, R. 2017 Computation of the starting vortex flow past a flat plate. *Proc. IUTAM* **20**, 136–143.