A dynamical method for the determination of Young's modulus by stretching. By C. F. SHARMAN, M.A., King's College.

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There are two general methods of measuring the elastic constants of bodies; one involves a study of the static deformation produced by the appropriate kind of stress, and the other a measurement of the period of oscillation of a system of known inertia under the elastic forces.

Young's modulus is directly concerned with two fairly simple types of strain, namely, stretching and bending; and the combination of these with the two methods of observation mentioned above provides four ways of measuring the constant. Three of the methods are quite well known*, and as applied to metal wires are used in the Cavendish Laboratory as class experiments. The simplest arrangement for the dynamical stretching method consists of a long, thin vertical wire loaded to a point well within the elastic limit by a mass attached to the lower end; observations are made of the period of vertical oscillation of the mass. There are a number of practical objections to this method:

(*a*) In order to obtain synchronous vibrations the extension of the wire must always be below the elastic limit. This means that for a metallic wire of reasonable length and cross-section the maximum amplitude is too small for convenient visual observation.

(*b*) The period of such a system is very short since the inertia is limited by the tensile strength of the wire. This objection applies principally to metals, where Young's modulus is very large.

(*c*) Owing to the small inertia of the vibrating system the total energy is small and the hysteresis losses in the wires are usually sufficient to bring the system to rest after a fraction of a second.

(*d*) Special precautions must be taken against yielding of the support under the large periodic forces.

In order to eliminate the objections (*b*) and (*c*) it is necessary to arrange that the inertia of the system and the tension in the wire are separately variable. The object of the present note is to describe a method where this has been achieved, and where the other disadvantages of the simple arrangement are much less serious.

* See *Experimental Elasticity*, Dr G. F. C. Searle, F.R.S., Camb. Univ. Press, 1920.

In Fig. 1, *AB* is the body which supplies most of the inertia of the system; it consists of a rectangular iron bar (2 × 3 × 60 cm.) into the ends of which are screwed two lead cylinders coaxial with its long axis. It swings about a hardened steel knife-edge, set into it with a screw, so that the axis of the oscillations passes through, or very slightly below, the centre of mass. Hence under the action of gravity alone it is aperiodic. The knife-edge rests on two agate planes which are supported on the top cross-piece of the iron stand shown in the side elevation of Fig. 1.

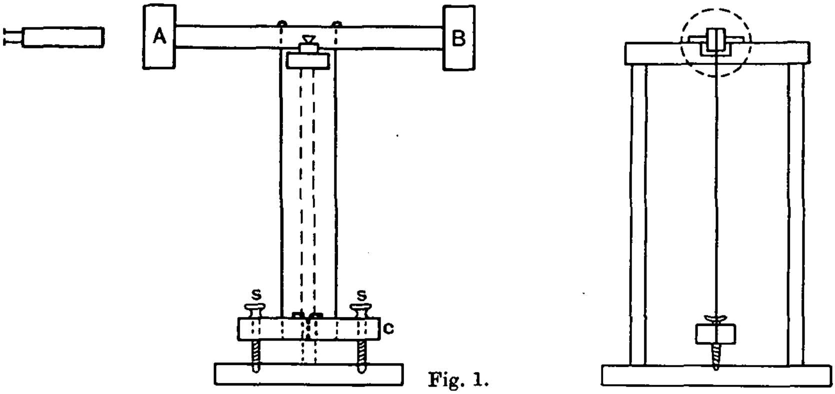

Fig. 1.

Two wires, each about 80 cm. long, are stretched between the vibrating bar and a heavy brass bar *C* which is screwed to the base of the stand by the large screws *S*, *S*. The wires are parallel, are separated by a distance of 4 cm., and are equidistant from the knife-edge. The tension in them is adjusted by means of the screws *S*, *S*. It is necessary that both ends of the wires should be rigidly anchored to the two bars: the methods shown in detail in Fig. 2 proved fairly satisfactory.

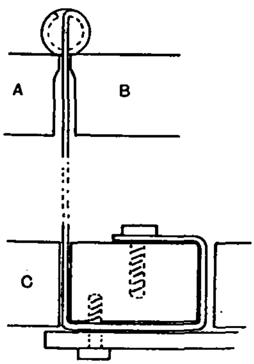

Fig. 2.

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With respect to the design of the inertia bar the following points are important:

(*a*) It must have a large moment of inertia—about 5.10^6 grm. cm.^{2} is convenient.

(*b*) Its total weight must not be so great as to injure the knife-edge or the agates.

(*c*) The bar must be stout enough to eliminate all possibility of bending under the forces exerted on it by the wires.

Let l be the normal length of the wires, ρ their radius of cross-section, E the Young's modulus of the material, r the distance from the knife-edge to the wires, and I the moment of inertia of the system. Then, for an angular displacement θ of the bar, we have from the definition of the modulus,

$$T/\pi\rho^2 = (r\theta/l)\,E.$$

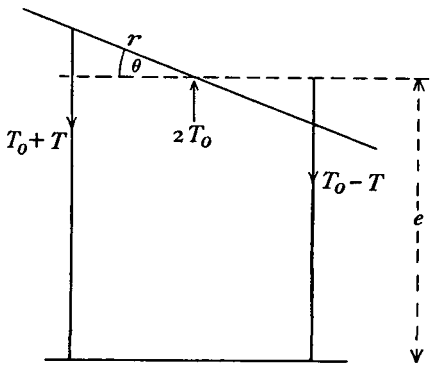

Fig. 3.

But the system of forces, which is represented in Fig. 3, is equivalent to a couple of moment $2Tr$.

Equating this to $Id^2\theta/dt^2$, we obtain the equation of motion of the system

$$2Tr = (2\pi\rho^2 r^2 E\theta)/l = Id^2\theta/dt^2.$$

This represents a simple harmonic oscillation of period

$$\tau = 2\pi\sqrt{\frac{Il}{2\pi\rho^2 r^2 E}},$$

whence

$$E = \frac{2\pi Il}{\tau^2\rho^2 r^2}.$$

From this result it is evident that special care is required in the measurement of the quantities τ, ρ, and r, all of which are squared.

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It is important that T shall never become equal to T_0, since it has been assumed in the theory that both wires are always taut. This condition is satisfied if a musical note can be obtained by plucking both wires when the bar is clamped in its position of maximum angular displacement.

Another important point is that the reaction at the knife-edge is $2T_0$ always, hence the elasticity of the stand does not enter into the result.

The oscillations were timed by observing a scratch on one of the lead cylinders through a low-power microscope. Even using lead wires observations could be taken over about a minute, but with hard steel a thousand vibrations taking a quarter of an hour were easily obtained.

The following table shows a few results obtained with common laboratory materials:

Material	Young's Modulus (dynes cm.$^{-2}$)
Steel (Banjo string)	$2{\cdot}07 \times 10^{12}$
Brass (Hard)	$1{\cdot}09 \times 10^{12}$
Aluminium	$0{\cdot}73 \times 10^{12}$
Copper (Annealed)	$1{\cdot}17 \times 10^{12}$
Nickel	$2{\cdot}18 \times 10^{12}$
Phosphor-Bronze	$1{\cdot}23 \times 10^{12}$

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