Rational normal octavic surfaces with a double line, in space of five dimensions: Addition. By Mr D. W. BABBAGE, Magdalene College.

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In a recent paper* in these *Proceedings* on the rational normal octavic surfaces with a double line in [5] I found four such surfaces, $F^8_{(1)}$, $F^8_{(2)}$, $F^8_{(3)}$, and $F^8_{(4)}$, representable on a plane respectively by the systems of curves $C^5(2^2, 1^9)$, $C^6(4, 1^{12})$, $C^6(2^6, 1^4)$, and $C^7(3, 2^8)$, with the base points in each case lying on an elliptic cubic. Inadvertently I overlooked a solution of certain indeterminate equations which leads to a fifth type $F^8_{(5)}$ represented by the plane system $C^9(3^8, 1)$.

The surface $F^8_{(5)}$ is the projection from a general point of itself of the surface F^9, normal in [6] and represented by $C^9(3^8)$, which is mentioned by L. Roth† in his recent paper on non-singular surfaces of sectional genus four. F^9 contains a pencil of elliptic cubic curves passing through a point O, and it projects from O into a double rational normal quartic scroll with a line directrix. F^9 is in fact the residual intersection of a point-cone V_3^4 in [6], whose cross-section is a quartic scroll of this type, with a cubic primal through three of its planes. The projected surface $F^8_{(5)}$ contains a pencil of elliptic cubics of which one member degenerates into a double line and a simple line. $F^8_{(5)}$ cannot be obtained as the intersection of quadrics, since any quadric through it will necessarily contain the rational point-cone V_3^3 generated by its cubics.

We can also obtain a normal surface F^9 in [6] of sectional genus four from the general rational plane-scroll V_3^4 by cutting this with a cubic primal through three of its planes. If we represent the prime sections of V_3^4 by quadric surfaces through a line in [3], F^9 will be represented by a cubic surface ϕ and its prime sections by quadric sections of ϕ through three collinear points; hence F^9 can be represented in a plane by the system $C^6(2^6, 1^3)$, where the nine base points form an associated set. If we project F^9 from a general point of itself we obtain a normal surface $F'^8_{(3)}$ in [5] with a double line. $F'^8_{(3)}$ may be regarded as a special case of our surface $F^8_{(3)}$ in that, like $F^8_{(3)}$, it is representable by the plane system $C^6(2^6, 1^4)$, though with the restriction that three of the simple base points form an associated set with the six double base points; but since

* Babbage, "Rational normal octavic surfaces with a double line, in space of five dimensions", *Proc. Camb. Phil. Soc.*, 29 (1933), 95–102.

† Roth, "On surfaces of sectional genus four", *Proc. Camb. Phil. Soc.*, 29 (1933), 184–194.

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it lies on a V_3^3 it cannot, as $F^8_{(3)}$ can, be obtained as the complete intersection of quadrics.

If we take the third type of V_3^4 in [6], which is a point-cone whose cross-section is a quartic scroll with ∞^1 conic directrices, and cut it by a cubic primal Φ through three of its planes, we obtain another surface F^9 of sectional genus four. Since the three planes are not coprimal, Φ will necessarily have a node at the vertex O of V_3^4; hence F^9 will have O as a quintuple point and will contain a pencil of plane cubics each with a node at O. This F^9 is not normal.

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