11

Modules

11.1 Modules

In this chapter, we discuss important *modules* such as Wollaston prisms, Newtonian prisms, beam splitters, phase shifters, mirrors, and null modules. Modules are pieces of apparatus placed at various strategic positions in the information void and are designed to influence labstate amplitudes at signal detectors. Modules are therefore a critical component in the quantized detector network (QDN) approach to quantum mechanics (QM).

Modules are not classified as either real or virtual detectors, because no information is extracted from them. Their function is solely to influence quantum amplitude propagation *between* stages in specific ways. Even empty space (the vacuum) can be regarded as a module, for the propagation of signals through nominally "empty" space is a fundamental subject in its own right. A particularly important example illustrating how nontrivial that can be is the Hubble–Doppler red shift of light from distant galaxies.

In the QDN description of the double-slit (DS) experiment, shown in Figure 10.2, the stage Σ_1 detectors labeled 1_1 and 2_1 may be considered as on the V_2 side of the wall W, where V_2 is the information void between stages Σ_1 and Σ_2.

Viewed in this way, the wall may be interpreted as a part of the apparatus that is positioned in the information void region V_1 so as to influence the signals obtained at detectors 1_1 and 2_1. Virtually all experiments have similar components of apparatus in the information void. We shall call any such piece of apparatus a *module*. Modules are necessary to the architecture of the experiment, are situated in the information void, and therefore are not detectors.

Examples of modules are Stern–Gerlach (SG) inhomogeneous field magnets, Wollaston prisms, mirrors, phase changers, and so on. Even empty space, otherwise known as the *vacuum*, should be regarded as a module.

Each module has its own physical properties. These include the number of detectors feeding amplitudes into that module and the number of detectors that

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amplitudes are fed out from. We shall discuss a number of modules relevant to the experiments discussed in this book, starting with the vacuum.

11.2 The Vacuum

In this book, we make a distinction between the *information void* concept and the *vacuum* of elementary particle physics. The differences are subtle.

Each concept has the characteristic that no signal detectors are associated with it. While it is commonplace to talk about "observers *in* empty space" or "detectors *in* the vacuum," such expressions are manifestly inconsistent if taken literally, even in classical mechanics (CM). What distinguishes the information void concept from the vacuum concept is that the latter is a mathematical objectivization of the former, based on some input context. For instance, if we believe that the information void has some structure or physical properties associated with a vacuum, then we may choose to believe in or set up some sort of mathematical model of empty space, such as a three-dimensional manifold with a Euclidean metric, as in Newtonian mechanics, or a general relativistic (GR) spacetime with a metric, or think of it in operator terms, as in Snyder's noncommuting spacetime (Snyder, 1947a,b), or even assign a quantum state vector to it, as in Fock space and relativistic quantum field theory (RQFT).

We are here faced with an important question as to the status of the vacuum: is it part of the relative external context (REC) that defines the observer, or is it part of the relative internal context (RIC) that defines the apparatus?

On the one hand, we have argued in earlier chapters that real observers are always *endophysical*, that is, are sitting inside the physical Universe. According to GR, the physical space that we image observers to be sitting in will have some physical attributes, such as metrical structure, curvature, mass, and energy densities. In this context, empty space plays a classical, auxiliary role as part of REC.

On the other hand, the vacuum concept will play a fundamental role in the calculation of quantum signal propagation amplitudes, such as black hole physics, where the classical background spacetime structure plays an important, even crucial, role (Birrell and Davies, 1982; DeWitt, 1975). In that context, the vacuum contributes to the RIC.

Elementary particle physics has been very successful in modeling the vacuum as having special relativistic (SR) symmetries and certain physical properties such as zero electric charge density, and so on. The standard model of particle physics appears not to need or use any concepts associated with the so-called quantum gravity program, a conjectural program based on the belief that the equations of GR and those of QM should be unified. The jury is out on all programs of research that attempt to give a detailed model of the vacuum, such as quantum gravity, string theory, and noncommutative geometry such as that of Snyder (Snyder, 1947a,b).

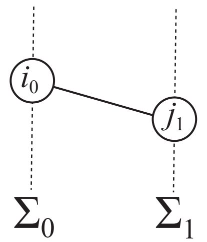

Figure 11.1. Amplitude propagation across the information void.

The question of what the best model of the "internal" vacuum of the RIC could be is a crucial one, because there are many contexts where the answer will influence the calculation of the quantum signal amplitudes of relevance to QDN. For example, we may well need to take spacetime curvature into account if we are sending signals across vast distances. Such a scenario is found in all observations in astronomy and cosmology.

We have indicated previously that QDN has nothing to say about the information void or the vacuum, precisely because QDN models what is happening at the signal detectors. Therefore, we will need to bring in standard physics theory, such as relativistic quantum field theory, to help us write down signal amplitudes between those detectors. Should that appear to make QDN rather limited, we point out that standard theories do have their limitations as well, such as the appearance of infinite renormalization constants in quantum field theory. We interpret that as the same problem seen from the opposite direction. It will probably be only by a judicious combination of QDN (or whatever should replace it) on the apparatus side of the physics coin and of standard relativistic quantum field theory on the reductionist side that we will find a better approach to empirical physics than with just either alone.

In QDN notation, signal amplitude propagation across the information void is represented by featureless lines, as in Figure 11.1.

11.3 The Wollaston Prism

A Wollaston prism is a quantum optics module that splits up a beam of light into two orthogonally polarized beams. Figure 11.2 is a schematic QDN diagram of such a device.

Consider an initial total state

$$|\boldsymbol{\Psi}_0) \equiv \left\{\alpha|s_0^1\rangle + \beta|s_0^2\rangle\right\} \otimes \mathbf{1}_0, \tag{11.1}$$

where $\mathbf{1}_0 \equiv \widehat{\mathbb{A}}_0^1 \mathbf{0}_0$ and$|s_0^i\rangle$ is a stage-Σ_0 polarization state vector in two-dimensional photon polarization Hilbert space $\mathcal{H}_0$. Here $i = 1$ or 2 represents either of two orthogonal polarizations such as *horizontal* and *vertical*. The coefficients α and β are complex numbers. If the initial state is normalized to unity, then $|\alpha|^2 + |\beta|^2 = 1$.

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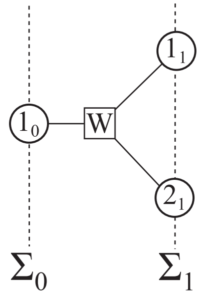

Figure 11.2. The Wollaston prism.

The contextual evolution operator[1] $\mathbb{U}_{1,0}$ from stage Σ_0 to stage Σ_1 is assumed semi-unitary, mapping from a two-dimensional contextual subspace of the initial four-dimensional Hilbert space $\mathcal{H}_0 \otimes Q_0^1$ to the final eight-dimensional one $\mathcal{H}_1 \otimes \mathcal{Q}_1$, where $\mathcal{H}_1$ is a copy of $\mathcal{H}_0$ and $\mathcal{Q}_1 \equiv Q_1^1 Q_1^2$. The assumed rules for the Wollaston prism are

$$\mathbb{U}_{1,0}\{|s_0^1\rangle \otimes \mathbf{1}_0\} = |s_1^1\rangle \otimes \mathbf{1}_1, \quad \mathbb{U}_{1,0}\{|s_0^2\rangle \otimes \mathbf{1}_0\} = |s_1^2\rangle \otimes \mathbf{2}_1, \tag{11.2}$$

where $\mathbf{1}_1 \equiv \widehat{\mathbb{A}}_1^1 \mathbf{0}_1$ and $\mathbf{2}_1 \equiv \widehat{\mathbb{A}}_1^2 \mathbf{0}_1$.

Using contextual completeness we have

$$\mathbb{U}_{1,0} = |s_1^1\rangle\langle s_0^1| \otimes \mathbf{1}_1\overline{\mathbf{1}_0} + |s_1^2\rangle\langle s_0^2| \otimes \mathbf{2}_1\overline{\mathbf{1}_0} \tag{11.3}$$

and its retraction

$$\overline{\mathbb{U}}_{1,0} = |s_0^1\rangle\langle s_1^1| \otimes \mathbf{1}_0\overline{\mathbf{1}_1} + |s_0^2\rangle\langle s_1^2| \otimes \mathbf{1}_0\overline{\mathbf{2}_1}. \tag{11.4}$$

These operators satisfy the semi-unitary relation

$$\overline{\mathbb{U}}_{1,0}\mathbb{U}_{1,0} = \{|s_0^1\rangle\langle s_0^1| + |s_0^2\rangle\langle s_0^2|\} \otimes \mathbf{1}_0\overline{\mathbf{1}_0} = I_0^{\mathcal{H}} \otimes \mathbb{I}_0^c, \tag{11.5}$$

the identity operator for the initial contextual total Hilbert space $\mathcal{H}_0 \otimes \mathcal{Q}_0^c$.

There are two generalized Kraus matrices associated with stage Σ_1, given by

$$\begin{aligned} M_{1,0}^1 &\equiv \overline{\mathbf{1}_1}\mathbb{U}_{1,0} = |s_1^1\rangle\langle s_0^1| \otimes \overline{\mathbf{1}_0}, \\ M_{1,0}^2 &\equiv \overline{\mathbf{2}_1}\mathbb{U}_{1,0} = |s_1^2\rangle\langle s_0^2| \otimes \overline{\mathbf{1}_0}, \end{aligned} \tag{11.6}$$

with retractions

$$\begin{aligned} \overline{M}_{1,0}^1 &= |s_0^1\rangle\langle s_1^1| \otimes \mathbf{1}_0, \\ \overline{M}_{1,0}^2 &= |s_0^2\rangle\langle s_1^2| \otimes \mathbf{1}_0. \end{aligned} \tag{11.7}$$

From these, the generalized POVM operators associated with Σ_1 are given by

$$\begin{aligned} E_{1,0}^1 &\equiv \overline{M}_{1,0}^1 M_{1,0}^1 = |s_0^1\rangle\langle s_0^1| \otimes \mathbf{1}_0\overline{\mathbf{1}_0}, \\ E_{1,0}^2 &\equiv \overline{M}_{1,0}^2 M_{1,0}^2 = |s_0^2\rangle\langle s_0^2| \otimes \mathbf{1}_0\overline{\mathbf{1}_0}. \end{aligned} \tag{11.8}$$

[1] As stated in a previous chapter, we drop the superscript "c" denoting "contextual."

In this particular case, these POVMs satisfy the relations $E^i_{1,0}E^j_{1,0} = \delta^{ij}E^i_{1,0}$ (no sum over i) and

$$E^1_{1,0} + E^2_{1,0} = \left\{|s^1_0\rangle\langle s^1_0| + |s^2_0\rangle\langle s^2_0|\right\} \otimes \mathbf{1}_0\overline{\mathbf{1}_0} = I_{\mathcal{H}} \otimes \mathbb{I}^c_0, \tag{11.9}$$

the identity operator for the initial contextual Hilbert space $\mathcal{H} \otimes \mathcal{Q}^c_0$. From these operators we find the conditional outcome rates

$$\Pr(\mathbf{1}_1|\mathbf{\Psi}_0) \equiv Tr\left\{E^1_{1,0}\boldsymbol{\varrho}_0\right\} = |\alpha|^2, \quad \Pr(\mathbf{2}_1|\mathbf{\Psi}_0) \equiv Tr\left\{E^2_{1,0}\boldsymbol{\varrho}_0\right\} = |\beta|^2, \tag{11.10}$$

assuming complete efficiency. Here $\boldsymbol{\varrho}_0 \equiv |\mathbf{\Psi}_0)(\mathbf{\Psi}_0|$.

11.4 The Newtonian Prism

Newton's researches in optics revealed features of light that demonstrate fundamental properties of relevance to us here (Newton, 1704). In this section we discuss two of his observations with prisms.

The Splitting of Light

Newton found that a beam of white light incident on a prism P^1 would be split into a *spectrum*, a set of emerging rays each of a different color, according to anyone looking at it.[2] If any one of those single-color component subbeams were in turn passed through another prism P^2, no further splitting occurred; Figure 11.3(a). From this, Newton concluded that the primary colors in white light were associated with properties of that light, and not induced by the prism. Figure 11.3(b) shows the same process as described by QDN.

Significantly, Newton took the view that the perception of color itself was a sensation induced in the mind by the processes of visual observation. His reasoning is based on empirical evidence: he noticed that superposing certain primary colors in the spectrum emerging from a prism created colors such as purple, in the mind of the observer, that were not contained in the original spectrum. In that respect he could reasonably be regarded as having a deeper view of observation than that generally associated with classical mechanics (CM), which pays no lip service to the observer and their subjective perceptions.

Newton's idea was vindicated subsequently by the development of the theory of color vision (the colors that humans believe they "see"), principally by Young and Helmholtz. They proposed that the human eye uses three distinct types of receptor (that is, detector). The signal information from these detectors is then processed to trigger the color sensations that we think we see.

The QDN modeling is based on the tensor product total Hilbert space $\mathcal{H}_n$ at stage Σ_n defined as $\mathcal{H}_n \equiv \mathcal{H}^{EM}_n \otimes \mathcal{Q}_n$, where $\mathcal{H}^{EM}_n$ is the standard RQFT

[2] Note that this expression is based on the ancient and misleading paradigm that observers look "*at*" objects.

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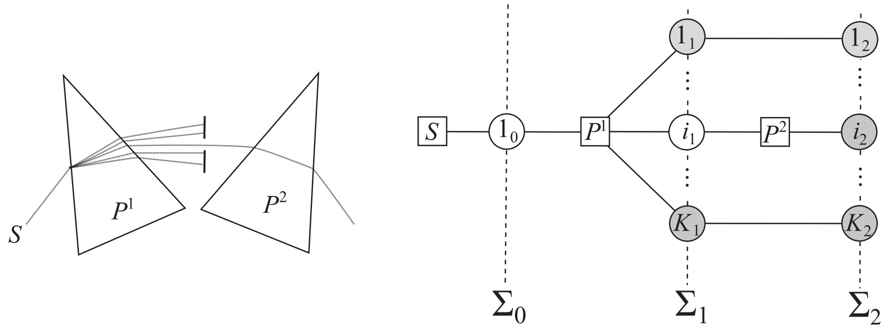

(a) (b)

Figure 11.3. (a) Newtonian prism P^1 splits an incoming beam of white light from source S (the Sun) into subbeams associated with primary colors. Prism P^2 does not split any one of those subbeams further. (b) The QDN schematic of the same process.

Hilbert space containing free photon states[3] and $\mathcal{Q}_n$ is the apparatus quantum register at that stage. Relevant states in $\mathcal{H}_n^{EM}$ are constructed in terms of a suitable orthonormal set $\{|i_n\rangle : i = 1, 2, \ldots K\}$, with orthonormality condition $\langle i_n | j_n \rangle = \delta^{ij}$.

With reference to Figure 11.3(b), the stage-Σ_0 normalized incident state $|\mathbf{\Psi}_0)$ is defined as

$$|\mathbf{\Psi}_0) \equiv \sum_{i=1}^{K} \psi_0^i |i_0\rangle \otimes \mathbf{1}_0, \qquad \sum_{i=1}^{K} |\psi_0^i|^2 = 1. \tag{11.11}$$

The action of the first prism P^1 in Figure 11.3 is taken to be

$$|\mathbf{\Psi}_0) \underset{P^1}{\rightarrow} |\mathbf{\Psi}_1) = \sum_{i=1}^{K} \psi_0^i |i_1\rangle \otimes \widehat{\mathbb{A}}_1^i \mathbf{0}_1, \tag{11.12}$$

which models the splitting up of the original beam into its primary colour constituents.

Three points to note here are the following.

Index Labels

The electromagnetic state indices i_0 and i_1 have a temporal subscript that labels stages only and does not affect the “value” of the index. So, for example, we take $i_1 = i_0 \equiv i$.

Discreteness

Contrary to what we might anticipate from experience with RQFT, the electromagnetic index i_n in the above is discrete, not continuous, because we can

[3] Any reference to *photons* is for convenience. The “basis” states here form a discrete set and therefore are not plane wave solutions but approximate them suitably.

only ever have a discrete set of detectors. This overturns the usual approach to RQFT, which presupposes a spectrum of photon states with a continuous frequency range. We do not need to deal with any supposed continuity in $\mathcal{H}_n^{EM}$, because the only parts of it that we can ever deal with are found at the finite, discrete detector end. Everything done here is contextual: it is the presupposition that an unobserved (and unobservable) continuum of states should play a role in, for example, Feynman graph integrals that contributes to the divergence of renormalization constants in RQFT. In the laboratory, all signals are finite.

The effect of the first prism P^1 on the incident beam is modeled by the action of a contextual semi-unitary evolution operator $U_{1,0}$, given by

$$U_{1,0} \equiv \sum_{i=1}^{K} |i_1\rangle\langle i_0| \otimes \widehat{\mathbb{A}}_1^i \mathbf{0}_1 \overline{\mathbf{1}_0}, \tag{11.13}$$

where K is the number of subbeams in the emerging spectrum that is relevant to the discussion. Newton chose K to be 7, referring to the subbeams as *red, orange, yellow, green, blue, indigo*, and *violet* (Newton, 1704). This choice is dependent on the observer, for if we had equipment that could detect infrared or ultraviolet light, we would have a different value for K.

Coherence versus Incoherence

When physical process are affected by either constructive or destructive interference of amplitude waves, there are two factors to take into account: the amplitudes should have the same frequency (more or less), and if so, they should be coherent. In the case of white light, neither of these factors is in play because by definition, white light consists of waves of many different frequencies, and these are necessarily incoherent. That does not mean that we cannot describe such situations with our QDN formalism. Newton's recombination experiment is a demonstration of incoherent superposition but not of constructive or destructive interference.

With reference to Figure 11.3(a), we see that only one chosen subbeam, labeled i in Figure 11.3(b), enters the second prism P^2, all the other subbeams being blocked off. Then we may write

$$U_{2,1} \left\{ |i_1\rangle \otimes \widehat{\mathbb{A}}_1^i \mathbf{0}_1 \right\} \equiv |i_2\rangle \otimes \widehat{\mathbb{A}}_2^i \mathbf{0}_2. \tag{11.14}$$

But what about the other, blocked subbeams?

There is a significant point here that we will be able to address more carefully in Chapter 25 and that has to do with information extraction versus *decommissioning*. By the term *decommissioning*, we mean the blocking off of signals and the elimination of the corresponding detectors from further consideration. We can see this in Figure 11.3(a), where all except one of the spectral subbeams emerging from P^1 are blocked off, with the remaining one allowed to pass through prism P^2.

The QDN approach to this issue is two-fold. In Figure 11.3(b), all of the stage-Σ_1 detectors except i_1 are shown shaded. This indicates that those components are either blocked off or observed irreversibly, with consequent information extraction. This blocking off/information extraction plays no further role in the experiment, and so is represented in the diagram by null tests taken from stage Σ_1 to stage Σ_2. On the other hand, Figure 11.3(b) indicates that i_1 sends a signal through the prism P^2 that is subsequently observed at i_2, which is now shown shaded in the figure.

Since the evolution from stage Σ_1 to stage Σ_2 is essentially trivial (which is what null tests are in practical terms), we may also write

$$U_{2,1}\left\{|j_1\rangle \otimes \widehat{\mathbb{A}}_1^j \mathbf{0}_1\right\} \equiv |j_2\rangle \otimes \widehat{\mathbb{A}}_2^j \mathbf{0}_2, \qquad j \neq i. \tag{11.15}$$

Hence we deduce

$$U_{2,1} = \sum_{\substack{j=1,\\ j\neq i}}^{K} \underbrace{|j_2\rangle\langle j_1| \otimes \widehat{\mathbb{A}}_2^j \mathbf{0}_2 \overline{\mathbf{0}_1} \mathbb{A}_1^j}_{\text{null tests}} + \underbrace{|i_2\rangle\langle i_1| \otimes \widehat{\mathbb{A}}_2^i \mathbf{0}_2 \overline{\mathbf{0}_1} \mathbb{A}_1^i}_{\text{through } P^2}. \tag{11.16}$$

Although the two terms on the right-hand side of this expression look similar, they are very different contextually. A practical difference would be that $|j_2\rangle$ is an exact copy (in $\mathcal{H}_2^{EM}$) of $|j_1\rangle$ (in $\mathcal{H}_1^{EM}$), for $j \neq i$, whereas $|i_2\rangle$ would not be an exact copy of $|i_1\rangle$ because of, say, attenuation effects as that subbeam passed through P^2.

The point here is that context underpins the significance of everything, and that context is determined by the observer. If they decide to block off all subbeams except the one labeled i, and then actually do that, then that action (not the decision alone) automatically brings in empirically significant context that involves separate dynamics and apparatus. It is the essence of Wheeler's participatory principle quoted on Chapter 1 that action be actually carried out when discussing quantum processes.

We see from this that the freedom that the observer has to choose how to rearrange apparatus creates a contextuality. This is emphasized in the next part of our description of the Newtonian prism, the recombination of a split spectrum back into the original beam.

The Recombination of Light

In his remarkable book *Opticks*, Newton discusses an extraordinary variant of the prism experiment shown in Figure 11.3(a) (Newton, 1704). He inverted prism P^2 and placed a lens between the two prisms. The lens refocused the spectrum emerging from P^1 onto P^2, as shown in Figure 11.4(a). The result was to recombine the spectrum, back into the original form of a beam of white light emerging from P^2.

Certainly, the recombined beam would not be precisely the same as the original, in that there would be some attenuation due to passing through glass, but as in the case of friction in mechanics, such effects can be regarded as secondary and not relevant to the main discussion.

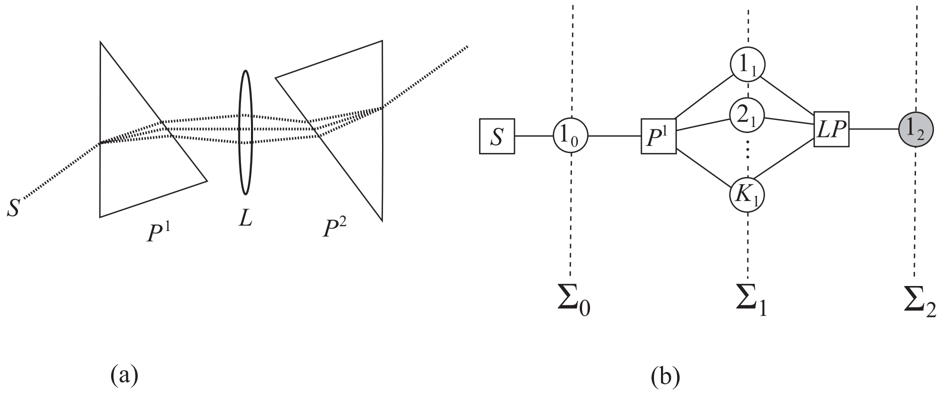

Figure 11.4. (a) Newton used lens L and prism P^2 to recombine the spectrum from prism P^1 back into a beam of white light. (b) In the QDN schematic of the same process, the lens and second prism constitute module LP.

The QDN stage diagram for Newton's recombination experiment is shown in Figure 11.4(b), with the following analysis.

Stage $\boldsymbol{\Sigma_0}$

The initial labstate is

$$|\boldsymbol{\Psi}_0) \equiv \sum_{i=1}^{K} \psi_0^i |i_0\rangle \otimes \mathbf{1}_0, \qquad \sum_{i=1}^{K} |\psi_0^i|^2 = 1. \tag{11.17}$$

Stage $\boldsymbol{\Sigma_0 \rightarrow \Sigma_1}$

The evolution operator $U_{1,0}$ that models the action of prism P^1 is given by

$$U_{1,0}^c = \sum_{i=1}^{K} |i_1\rangle\langle i_0| \otimes \widehat{\mathbb{A}}_1^i \mathbf{0}_1 \overline{\mathbf{1}_0}. \tag{11.18}$$

Hence the labstate $|\boldsymbol{\Psi}_1)$ emerging from prism P^1 is given by

$$|\boldsymbol{\Psi}_1) \equiv U_{1,0}|\boldsymbol{\Psi}_0) = \sum_{i=1}^{K} \psi_0^i |i_1\rangle \otimes \widehat{\mathbb{A}}_1^i \mathbf{0}_1. \tag{11.19}$$

Stage $\boldsymbol{\Sigma_1 \rightarrow \Sigma_2}$

The evolution operator $U_{2,1}$ that models the undoing action of the module LP consisting of the lens L and the prism P^2 is given, by inspection, by

$$U_{2,1} = \sum_{i=1}^{K} |i_2\rangle\langle i_1| \otimes \widehat{\mathbb{A}}_2^1 \mathbf{0}_2 \overline{\mathbf{0}_1} \mathbb{A}_1^i. \tag{11.20}$$

Here, we have chosen to represent the action of LP as refocusing the spectrum from P^1 onto labstate $\mathbf{1}_2 \equiv \widehat{\mathbb{A}}_2^1 \mathbf{0}_2$. The final outcome total state $|\boldsymbol{\Psi}_2)$ is therefore given by

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$$|\mathbf{\Psi}_2) \equiv U_{2,1}|\mathbf{\Psi}_1) = \sum_{i=1}^{K} \psi_0^i |i_2\rangle \otimes \mathbf{1}_2. \tag{11.21}$$

Comparing this with the original labstate (11.17), we see that the final total state $|\mathbf{\Psi}_2)$ is a persistent image of $|\mathbf{\Psi}_0)$. Therefore, the combination of prism P^1 followed by prism P^2 is equivalent to a null test of the original beam.

It might be believed that we have encountered an example of semi-unitary evolution where the initial quantum register had a larger rank than the final quantum register. In fact, that is not the case. The action of the lens between the prisms is designed to focus on a one-dimensional subspace of the stage-Σ_1 register, a subspace containing the image of the original beam. It is that subspace alone that is then mapped by the lens and second prism into the one-dimensional subspace representing the outgoing recombined beam.

When Newton's recombination experiment is examined in fine detail, only then can it be appreciated how extraordinarily difficult it is to model what happens with some degree of correctness. The reader is invited to model this experiment using only quantum electrodynamics (QED) in its standard formulation, and they will then see what we mean.

11.5 Nonpolarizing Beam Splitters

Despite the implication in its name, a beam splitter will in general have two input channels and two outcome channels, as shown in Figure 11.5. In fact, it is perhaps best to think of a beam splitter as a specific example in quantum optics of a more general two-two particle scattering process that satisfies certain properties, such as unitarity and various conservation laws, such as conservation of energy, momentum, and charge.

In applications, we shall be interested in coherent, signality-one, normalized initial labstates $|\mathbf{\Psi}_0)$ involving both channels 1_0 and 2_0, of the generic form

$$|\mathbf{\Psi}_0) \equiv \psi^1 |s_0^1\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0 + \psi^2 |s_0^2\rangle \otimes \widehat{\mathbb{A}}_0^2 \mathbf{0}_0, \tag{11.22}$$

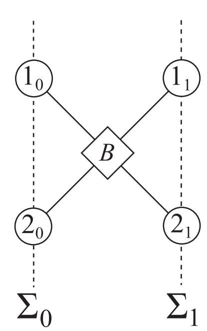

Figure 11.5. The beam splitter.

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where $|\psi^1|^2 + |\psi^2|^2 = 1$, and $|s_0^i\rangle$, $i = 1, 2$, are initial, normalized one-photon states, not necessarily orthogonal. In this scenario, the polarization properties associated with each input channel are unspecified, so the discussion here is rather general in that respect.

The dynamics is specified by stating the beam splitter rules for each input channel. In general, semi-unitary evolution applies (if we ignore inefficiency and dissipation), so we write

$$\begin{aligned} U_{1,0}\left\{|s_0^1\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0\right\} &= \alpha|s_1^1\rangle \otimes \widehat{\mathbb{A}}_1^1 \mathbf{0}_1 + \beta|s_1^1\rangle \otimes \widehat{\mathbb{A}}_1^2 \mathbf{0}_1, \\ U_{1,0}\left\{|s_0^2\rangle \otimes \widehat{\mathbb{A}}_0^2 \mathbf{0}_0\right\} &= \gamma|s_1^2\rangle \otimes \widehat{\mathbb{A}}_1^1 \mathbf{0}_1 + \delta|s_1^2\rangle \otimes \widehat{\mathbb{A}}_1^2 \mathbf{0}_1, \end{aligned} \tag{11.23}$$

where the beam splitter coefficients satisfy the semi-unitary relations

$$|\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1, \qquad \alpha^*\gamma + \beta^*\delta = 0. \tag{11.24}$$

It will be useful now and later to define the matrix of coefficients

$$B \equiv \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}. \tag{11.25}$$

Then the semi-unitary relations (11.24) tell us that B is in fact a unitary matrix.

It is easy to prove from (11.24) that $|\alpha| = |\delta|$ and $|\beta| = |\gamma|$, so we may write $\alpha \equiv ue^{iA}$, $\beta \equiv ve^{iB}$, $\gamma \equiv ve^{iC}$, and $\delta \equiv ue^{iD}$, where $|u|^2 + |v|^2 = 1$ and the phases A, B, C, and D are real. Now define the phase E by $2E \equiv A + D$. Then it is straightforward to show that B can be written in the form

$$B \equiv e^{iE} \begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}, \tag{11.26}$$

where $|a|^2 + |b|^2 = 1$ and the phase arguments of a and b are linear combinations of A, B, and C. We shall refer to the right-hand side of equation (11.26) as the *standard form* of a 2×2 unitary matrix.

Usually, the phase E in the standard form may be ignored. As for the complex coefficients a and b, these are related by Fresnel's equations for the reflection and transmission of light through optical media. Suppose a monochromatic beam of light passes through medium μ^1 and is incident on a plane surface boundary of medium μ^2. Generally, it will split into a reflected part that goes back into medium μ^1 and a transmitted part that moves into μ^2. If the speed of light c^1 in μ^1 is greater than the speed of light c^2 in μ^2, then the reflected part will be out of phase with the incident beam by π, that is, equivalent to a sign change. There is no such change in the phase of the transmitted part. If, conversely, c^1 is less than c^2, then there are no phase changes in either reflected or transmitted parts.

With this information there are two forms we may choose for B:

Form 1

$$B = \begin{bmatrix} t & -r \\ r & t \end{bmatrix}, \tag{11.27}$$

where t and r are real and satisfy the equation $t^2 + r^2 = 1$.

Form 2

$$B = \begin{bmatrix} t & ir \\ ir & t \end{bmatrix}, \tag{11.28}$$

where t and r are real and satisfy the equation $t^2 + r^2 = 1$.

In stage diagrams such as Figure 11.5, where the square representing the beam splitter is rotated as shown to face the two input ports 1_0 and 2_0, we shall adopt the convention that 2_1 and 1_1 are the transmitted and reflected beams, respectively, associated with input port 1_0, while they are the reflected and transmitted beams, respectively, associated with input port 2_0. This convention is used with Form 2 above throughout this book in the encoding of our computer algebra program MAIN, discussed in the next chapter.

In many experiments discussed in the literature, Form 2 is assumed, with t and r taken equal, that is, $t = r = 1/\sqrt{2}$. In our computer algebra program MAIN discussed in the next chapter, we do not do this. We use Form 2, but it is generally most instructive to leave the transmission and reflection coefficients arbitrary up to the required constraints. For instance, t^i and r^i will be the transmission and reflection coefficients for beam splitter B^i and will be arbitrary, apart from the condition that $(t^i)^2 + (r^i)^2 = 1$.

Signality-Two Input

On occasion, the possibility arises that a signality-two labstate or its equivalent is incident on a beam splitter. We shall refer to this as *beam splitter saturation*. The dynamics for such a scenario has to be decided contextually. One possibility is that the two photons (speaking loosely) do not emerge and are absorbed by the device. In such a case, the output channels of the beam splitter remain in their ground states. On the other hand, we could decide to model what happens as a case of *transparency*, such that the output labstate is a signality-two state with both output detectors registering a signal.

Exercise 11.1 Suppose a calibrated beam splitter satisfies the semi-unitary dynamics

$$\begin{aligned} \mathbb{U}_{1,0}\mathbf{0}_0 &= \mathbf{0}_1, \\ \mathbb{U}_{1,0}\widehat{\mathbb{A}}_0^1\mathbf{0}_0 &= \alpha\widehat{\mathbb{A}}_1^1\mathbf{0}_1 + \beta\widehat{\mathbb{A}}_1^2\mathbf{0}_1, \\ \mathbb{U}_{1,0}\widehat{\mathbb{A}}_0^2\mathbf{0}_0 &= \gamma\widehat{\mathbb{A}}_1^1\mathbf{0}_1 + \delta\widehat{\mathbb{A}}_1^2\mathbf{0}_1, \end{aligned} \tag{11.29}$$

where α, β, γ, and δ satisfy the semi-unitary relations (11.24).

Show that if

$$\mathbb{U}_{1,0}\widehat{\mathbb{A}}_0^1\widehat{\mathbb{A}}_0^2\mathbf{0}_0 = a\mathbf{0}_1 + b\widehat{\mathbb{A}}_1^1\mathbf{0}_1 + c\widehat{\mathbb{A}}_1^2\mathbf{0}_1 + d\widehat{\mathbb{A}}_1^1\widehat{\mathbb{A}}_1^2\mathbf{0}_1, \tag{11.30}$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ and $\mathbb{U}_{1,0}$ is semi-unitary, then $a = b = c = 0$ and $|d|^2 = 1$.

11.6 Mirrors

Mirrors are important modules in many experiments. The basic action of a mirror is to deflect electromagnetic radiation, that is, change path direction.

Depending on the physics, mirrors can also change the phase of electromagnetic waves. According to Fresnel's laws of optics, light incident from air on a mirror undergoes a phase shift of π, if reflecting from the front surface of a mirror, while there is no phase shift on rear surface reflection.

Naturally occurring mirrors can have a severe influence on signal detection amplitudes. For example, television signals received at an aerial are built up from the superposition of all the signals that have followed separate paths from the transmitting station to that aerial. If a transmitted signal has one path that goes in line of sight from transmitter to detector and another path that goes from transmitter, is reflected from the surface of the sea, and then arrives at the detector, then destructive interference can occur, thereby degrading the overall detected signal (Laven et al., 1970). Moreover, if the sea level changes due to tidal action, then the interference varies during the day and cannot be eliminated easily.

In stage diagrams, mirrors are labeled M.

11.7 Phase Changers

Mirrors are a specific example of more general modules referred to as *phase changers*. Such modules are fundamental to experiments where quantum interference is used to explore material properties, such as in interferometry.

Although phase changers can be used in any context, an important scenario involves electromagnetic signals, because in classical optics, Maxwell's equations lead to the conclusion that light is a wave process involving transversely oscillating electric and magnetic waves. Phase changer modules other than mirrors will be labeled by the phase angle involved.

11.8 Polarization Rotators

Electromagnetic waves have transverse electric and magnetic field polarization degrees of freedom, and this plays a significant role in many experiments. Generally, the convention is to define the polarization of a plane polarized electromagnetic wave as that of the electric field component. By default, the magnetic polarization is orthogonal to the electric field polarization.

We shall encounter some experiments where a polarized electromagnetic wave passes through a module that turns the wave's polarization plane by a known angle. Such a module will be labeled R^{θ} in stage diagrams, where θ is the angle of rotation.

11.9 Null Modules

A null module is any process that essentially does nothing to a signal. Because of this, a signal that is passed through a succession of null modules can be thought of as "on hold" until such time as the observer decides to look.

Null modules are used to model the concept of *persistence*, discussed in Chapter 18. On that account, null modules should not be considered trivial in the sense of identity operators, the action of which does nothing observable. In contrast, null modules require the right context and reflect a fundamental property of physics: that structures can persist over time in an observable sense.

Given a rank-r quantum register $\mathcal{Q}_n$ at stage Σ_n, a computational basis representation of a complete null operator (one that acts on the whole register) $\mathbb{N}_{n+1,n}$ that maps into stage Σ_{n+1} register $\mathcal{Q}_{n+1}$ of the same rank is

$$\mathbb{N}_{n+1,n} = \sum_{i=0}^{2^r-1} \boldsymbol{i}_{n+1}\overline{\boldsymbol{i}_n}. \tag{11.31}$$

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