On the substitution of Wallis's postulate of similarity for Euclid's postulate of parallels. Addendum. By Professor M. J. M. HILL, Peterhouse.

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The following is an alternative to Proposition II of the former paper*. It does not assume the second of the initial assumptions in Art. 2, nor does it assume that the angles supplementary to equal angles are equal, which Hilbert regards as a proposition (*Foundations of Geometry*, p. 18 of the English translation).

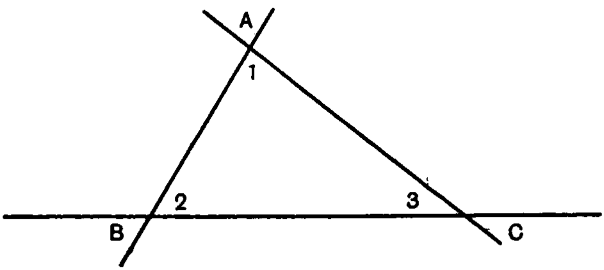
The assumptions in the following proof are:

(i) Through any two points one and only one straight line can be drawn.

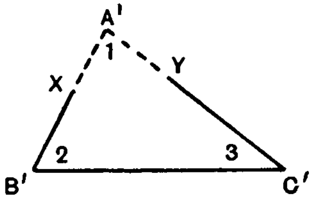
(ii) At any point a straight line can be drawn making with a given straight line through that point on a given side an angle equal to a given angle.

(iii) In the triangle ABC a straight line through A in the angle BAC must meet BC between B and C.

(iv) and (v) are included in Wallis's Postulate of Similarity and may be stated thus:



Let there be three straight lines in a plane which intersect so as to form a triangle ABC. Take any two points B', C'. At B' make the angle $C'B'X$ equal to the angle CBA. At C' make the angle $B'C'Y$ in the same plane as $C'B'X$ equal to the angle BCA.



Then it is assumed

(iv) that $B'X$ and $C'Y$ will meet at some point A', and

(v) that the angle $B'A'C'$ will be equal to the angle BAC.

* *Proc. Camb. Phil. Soc.* vol. 22 (1925), pp. 964–9.

It will now be shown that if a be a straight line in a plane, and A a point in the plane, but not on a, then it is possible to draw only one straight line through A in the plane, which does not meet a.

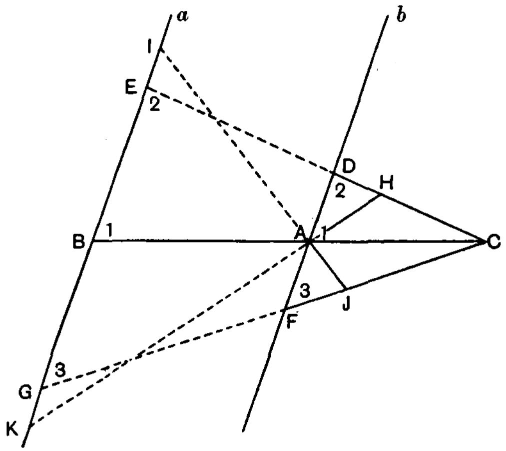
Join A to any point B on a. Produce BA through A to any point C.

Then through A a straight line b can be drawn making with BAC an angle equal to the angle which a makes with BAC.

By Proposition I, b does not meet a.

It will now be shown that every straight line through A, other than b, must meet a.

On b take any point D on the same side of BAC as that on which the equal angles bAC, aBC are situated; and join C to D.



By Art. 4, Corollary (ii), since the angles CAD, CBa are equal, it follows that CD, if produced, must meet a in some point E, and the angles CDA, CEB will be equal, since the triangles CAD, CBE are, by assumptions (iv) and (v) above, equiangular.

On b take any point F on the opposite side of A to D, and join CF.

Then since the angles CDF, CEB are equal, it follows by Art. 4, Corollary (ii) that CF, if produced, will meet a in some point G, and the angles CFD, CGE will be equal, since the triangles CFD, CGE are, by assumptions (iv) and (v) above, equiangular.

Now any straight line through A, other than b, must pass through either the angle CAD or the angle CAF. If it pass through the angle CAD it must meet CD in some point H between C and D.

Then since the angles HDA, HEB are equal, it follows by Art. 4, Corollary (ii) that HA must, if produced, meet a in some point K.

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If, however, a straight line through A pass through the angle CAF it must meet CF in some point J between C and F.

Then since the angles JFA, JGB are equal, it follows by Art. 4, Corollary (ii) that JA must, if produced, meet a in some point I.

Consequently every straight line through A, other than b, meets a.

Note. The triangles CFA, CGB have two angles of the one respectively equal to two angles of the other.

Therefore by Art. 4, Corollary (i) the remaining angles CAF, CBG are equal. These are the angles supplementary to the equal angles CAD, CBE.

This result is proved by Hilbert (*Foundations of Geometry*, p. 18) by a proof in which the congruence of triangles is used.